WALSH'S NEW GRAMMAR SCHOOL ARITHMETIC

PART TWO

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NEW

GRAMMAR SCHOOL ARITHMETIC

PART II

BY

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INTRODUCTION.

The New Grammar School Arithmetic forms with the New Primary Arithmetic a complete course in elementary school mathematics.

Each of the first four chapters of the New Grammar School Arithmetic provides for a half year, beginning with advanced matter, which is followed by a review and an extension of the topics of the preceding grades. Each of the next two chapters (V and VI) contains arithmetic work for a year, which should be supplemented by portions of the algebraic and geometrical material of Chapters VII and VIII. It is recommended that at least a portion of the work in equations of Chapter VII should precede the study of Chapter V.

Among the special features of the New Grammar School Arithmetic are the number and the variety of the problems; the systematic reviews, which cover oral and written drill work even in the fundamental operations; the attention paid to short, direct, business methods of computation; and the spiral handling of the various topics.

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SUGGESTIONS TO TEACHERS.

Additions and Omissions. — The teacher should freely supplement the work of the text-book when she finds it necessary to do so; and she should not hesitate to leave a topic that her pupils fully understand, even though they may not have worked all the examples given in connection therewith.

Oral and Written Work. — The heading "Written Problems" is merely a general direction, and it should be disregarded by the teacher when the pupils are able to do the work "mentally." The use of the pencil should be required only so far as it may be necessary. It is a pedagogical mistake to insist that the brighter pupils of a class should set down a number of figures that they do not need. As an occasional exercise, the pupils may be directed to give all the work required to solve a problem, and to make a written explanation of each step in the solution; but it should be the teacher's aim to have the majority of the examples done with as great rapidity as is consistent with absolute correctness. It will be found that, as a rule, the quickest workers are the most accurate.

Conduct of the Recitation. — It is often advisable, for some purposes, to divide an arithmetic class into two sections, even where the pupils are nearly equal in attainments. The members of one section may work examples from their books while the others write the answers to oral problems given by the teacher, etc.

Where a class is thus taught in two divisions, the members of each should sit in alternate rows, extending from the front of the room to the rear. Seated in this way each pupil is doing a different kind of work from those on the right and the left, and he does not have the temptation of a neighbor's work to lead him to compare answers.

To save time, explanations of new subjects may be given to the whole class; but much of the arithmetic work should be done in "sections," one of which is under the immediate direction of the teacher, while the other is employed in "seat" work. The "seat" work of pupils of the more advanced classes should consist largely of problems solved without assistance. Especial pains have been taken to grade the

problems so as to have none beyond the capacity of the average pupil. It is not necessary that all the members of a division should work the same problems at a given time, or the same number of problems, or that a new topic should be postponed until all of the previous problems have been solved.

Whenever it is possible, each of the members of the division working under the teacher's immediate direction should take part in all the work done. In mental arithmetic, for instance, while only a few may be called upon for explanations, all of the pupils should write the answers to each question. The same is true of much of the sight work, the approximations, some of the special drills, etc.

Drills and Sight Work. — To secure reasonable rapidity, it is necessary to have regular and systematic drills. These should be employed frequently, but should not last longer than five or ten minutes. A page of special sight drills is given in each chapter. These may also be used in oral problems.

It often happens that as pupils go forward in school they lose much of the readiness in oral and written work that they possessed in the lower grades, owing to the neglect of their teachers to continue to require quick, accurate review work in the operations previously taught. In this book these special drills follow the plan of the combinations of the earlier book, but gradually grow more difficult. They should first be used as sight exercises, either from the books or from the blackboard.

To secure valuable results from drill exercises, the utmost promptness in answers should be required.

Language. — While the use of correct language should be insisted upon in all lessons, children should not be required in arithmetic to give all answers in "complete sentences." Especially in the drills, it is important that the results be expressed in the fewest possible words. The teacher should be careful always to employ exact arithmetical language and to require it from the pupils.

Objective Illustrations. — The chief reason for the use of objects in the study of arithmetic is to enable pupils to work without them. While counters, weights and measures, diagrams, or the like are necessary at the beginning of some topics, it is important to discontinue their use as soon as the pupil is able to proceed without their aid.

Approximate Answers. — An important drill is furnished in the "approximations" (see Arts. 104, 180, 233, etc.). Pupils should be required in much of their written work to estimate the result before beginning to solve a problem with the pencil. Besides preventing an

absurd answer, this practice will also have the effect of causing a pupil to see what processes are necessary. In too many instances, work upon a problem is commenced before the conditions are grasped; this will be less likely to occur in the case of one who has carefully "estimated" the answer. The pupil will frequently find, also, that he can obtain the correct result without using his pencil.

Indicating Operations. — It is a good practice to require pupils to indicate by signs all of the processes necessary to the solution of a problem, before performing any of the operations. This frequently enables a pupil to shorten his work by cancellation, etc. In the case of problems whose solution requires tedious processes, some teachers do not require their pupils to do more than to indicate the operations. It is to be feared that much of the lack of facility in adding, multiplying, etc., found in the pupils of the higher classes is due to this desire to make work pleasant.

Sight Exercises. — Many pupils who find it difficult to solve problems read to them readily make the necessary calculations without a pencil when they have the numbers before them on the blackboard, or in their books. It may be found advisable to have a class first solve the whole of a given set of oral problems from their books, and at a later lesson write the answer to each question after it has been read by the teacher. In the case of sight exercises too difficult to be solved mentally, the set might be taken up one at a time by individual pupils, after which the pupils might be required to write answers "at sight" at a signal from the teacher. If the exercises are on the blackboard, the teacher might use a pointer to indicate the example whose answer was desired, not following the order in which they appeared on the blackboard. A similar method might be employed in sight work done from the books.

CHAPTER V.

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PERCENTAGE.

301. Preliminary Exercises.

Per cent means hundredths. Seven per cent means seven hundredths, $\frac{7}{100}$, or .07. It is written 7%.

How many hundredths of a number is its half? $\frac{1}{4}$ = how many hundredths? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{3}{4}$? $\frac{2}{5}$?

What per cent of a number is the half of it? $\frac{1}{4}$? $\frac{1}{3}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{20}$? $\frac{1}{25}$? $\frac{1}{50}$? $\frac{1}{100}$? $\frac{1}{200}$?

302. What per cent of a number is $\frac{2}{3}$ of it? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{5}{8}$? $\frac{7}{10}$? $\frac{3}{20}$? $\frac{2}{25}$? $\frac{7}{50}$? $\frac{3}{100}$?

303. 1 per cent of a number is equal to what fraction of it? 3%? 5%? 9%? 10%? 15%? 20%? 25%? 30%? 40%? 50%? 60%? 75%? 90%?

304. What fractions are equal to the following?

$12\frac{1}{2}\%$? $16\frac{2}{3}\%$? $33\frac{1}{3}\%$? $37\frac{1}{2}\%$? $6\frac{1}{4}\%$? $62\frac{1}{2}\%$? $66\frac{2}{3}\%$? $87\frac{1}{2}\%$? $\frac{1}{2}\%$? $\frac{1}{4}\%$? $2\frac{1}{2}\%$? $\frac{1}{8}\%$?

305. 3 times a number is what per cent of it? $2\frac{1}{2}$ times? $1\frac{1}{4}$ times? $4\frac{1}{5}$ times?

306. Oral Exercises.

1. Find $37\frac{1}{2}\%$ of $\$24$.

$37\frac{1}{2}\%$ of $\$24 = \frac{3}{8}$ of $\$24$, or $\$9$. *Ans.* $\$9$.

2. 6% of 150 bushels.

1% of 150 bushels $= 1.5$ bushels $= 1\frac{1}{2}$ bushels; and 6% is 6 times $1\frac{1}{2}$ bushels, or 9 bushels. *Ans.* 9 bushels.

3. 81% of 300 horses.

81% of 100 horses $= 81$ horses; of 300 horses it is 3 times 81 horses, or 243 horses. *Ans.* 243 horses.

In examples 2 and 3 the pupil should be led to see that he can point off two decimal places in the multiplicand instead of in the multiplier; without changing the result. The above analyses are suggestive merely. The form given in the third example is to furnish an explanation for the use of 3 as a multiplier.

4. Find $37\frac{1}{2}\%$ of 1 gallon.

$37\frac{1}{2}\%$ of 1 gal. $= \frac{3}{8}$ gal. $= 3$ pt. $= 1$ qt. 1 pt., *Ans.*

5. Find $12\frac{1}{2}\%$ of 1 gallon
6. $37\frac{1}{2}\%$ of $\$24$
7. $33\frac{1}{3}\%$ of 81 cows
8. 6% of 150 pounds
9. 4% of 125 bushels
10. $62\frac{1}{2}\%$ of 1 peck
11. $4\frac{1}{2}\%$ of $\$200$
12. 99% of 200 gallons
13. $\frac{1}{2}\%$ of $\$640$
14. $\frac{1}{4}\%$ of 800 yards
15. $66\frac{2}{3}\%$ of 66 horses
16. $16\frac{2}{3}\%$ of 1 yard
17. 81% of $\$300$
18. $2\frac{1}{2}\%$ of 80 sheep
19. 40% of $\$2.50$
20. 20% of 65 rods
21. 10% of 15 pounds
22. $3\frac{1}{3}\%$ of $\$60$
23. $\frac{1}{8}\%$ of $\$72$
24. $1\frac{1}{4}\%$ of $\$96$

The skilful teacher will appreciate the importance of rapid work, and will gradually shorten the time to be given to a class for the solution of an oral example. She will also vary her methods of conducting the recitation, so as to keep up the interest of the pupils.

TO FIND THE PERCENTAGE.

307. Written Exercises.

1. Find 6% of $91.50.

Multiply the *base*, $91.50, by the *rate*, 6, expressed as hundredths. The result, $5.49, is called the *percentage*.

$91.50
× .06
$5.4900 *Ans.*

To find the percentage, multiply the base by the rate expressed as hundredths.

2. $33\frac{1}{3}\%$ of $28.80.

While the rule is the same, to multiply $28.80 by $.33\frac{1}{3}$, the pupil should not fail to change $33\frac{1}{3}$ hundredths to one-third.

3)$28.80
$9.60 *Ans.*

3. $\frac{1}{8}\%$ of $1240.

$\frac{1}{8}\% = \frac{1}{800}$. Divide by 800 by cancelling the two ciphers in the divisor and making two decimal places in the dividend.

800)$12.40
$1.55 *Ans.*

4. $4\frac{1}{2}\%$ of $92.40.

$92.40 × .04\frac{1}{2}$.

5. 450% of $92.40.

$92.40 × 4.5.

6. 12% of $37.50
7. 20% of $51.60
8. 1400% of $89.70
9. $12\frac{1}{2}\%$ of $73.28
10. $13\frac{1}{3}\%$ of $27.60
11. $6\frac{1}{4}\%$ of $25.60
12. $3\frac{1}{3}\%$ of $47.40
13. $5\frac{1}{2}\%$ of $29.50
14. 860% of $38
15. $\frac{1}{8}\%$ of $2496
16. 25% of $52.36
17. 60% of $33.30
18. 8% of $19.50
19. $6\frac{2}{3}\%$ of $47.40
20. 12% of $62.50
21. $4\frac{1}{2}\%$ of $71.50

22. 40% of $28.30
23. 160% of $39.40
24. 84% of $23.75
25. $66\frac{2}{3}$% of $825
26. 75% of $59.20
27. $87\frac{1}{2}$% of $392
28. $93\frac{3}{4}$% of $496
29. $\frac{15}{16}$% of $496

Suggestion. — The teacher should have a preliminary sight lesson on these examples before giving them out for written solution.

TO FIND THE BASE OR THE RATE.

308. Preliminary Exercises.

1. 40 is one-half of what number?
2. 40 is .5 of what number?
3. 40 is 50% of what number?
4. 40 is what part of 80?
5. 40 is what decimal of 80?
6. 40 is how many hundredths of 80?
7. 40 is what per cent of 80?
8. 26 is what per cent of 65?

26 is $\frac{26}{65}$ of 65. The fraction $\frac{26}{65}$ equals $\frac{2}{5}$, or 40 hundredths. *Ans.* 40 per cent.

9. 26 is 40 per cent of what number?

If 40 hundredths of a number is 26, the number equals 26 divided by 40 hundredths, or 26 ÷ .40. *Ans.* 65.

To find the base, divide the percentage by the rate expressed as hundredths. To find the rate, divide the percentage by the base, expressing the result in hundredths.

Another method of finding the *base* or the *rate* is suggested in the illustrative examples on the next page, which give young pupils an introduction to the equation, a powerful instrument in mathematical investigation.

309. Written Exercises.

1. What per cent of 65 is 26?

This means, how many hundredths of 65 will equal 26? which may be expressed in the following form:

$$65 \times \frac{?}{100} = 26.$$

The *rate* being required, the foregoing may be written as follows:

$$65 \times \frac{r}{100} = 26, \text{ or } \frac{65\,r}{100} = 26.$$

This is called an *equation*. To solve the equation, that is, to obtain the value of r, the general method is to clear the equation of the fraction by multiplying both sides by the denominator of the fraction, 100. This gives $65\,r = 2600$, or 65 times r equals 2600. r, therefore, is equal to 2600 divided by 65. *Ans.* 40 per cent.

Proof. — 65×40 hundredths $= 26$.

2. 40 per cent of what number equals 26?

$$b \times \frac{40}{100} = 26, \text{ or } \frac{40\,b}{100} = 26.$$

Clearing of fractions, $40\,b = 2600$; $b = 65$, *Ans.*

Proof. — $40\,\%$ of $65 = 65 \times .40 = 26$.

3. 75 per cent of a number is 42. What is the number?

$$75\,\% = \tfrac{3}{4}.$$

$$b \times \frac{75}{100} = 42, \text{ or } \frac{3\,b}{4} = 42.$$

Clearing of fractions, $3\,b = 168$; $b = 56$, *Ans.*

4. What number is 15 per cent of 84?

$p = 15$ hundredths of 84.

5. 24 is 18 per cent of what number?

6. 27 per cent of a number is 81. What is the number?

7. A boy spelled correctly 20 words of 25 given out. What per cent of the words did he spell correctly?

NOTE. — 25 is the base, 20 is the percentage; required the rate.

8. 132 is 120 per cent of what number?

9. $\frac{1}{8}$ per cent of a number is 23. What is the number?

10. $\frac{3}{5}$ = what per cent of $\frac{4}{5}$?

$$\frac{4}{5} \times \frac{r}{100} = \frac{3}{5}. \quad \text{Cancelling, } \frac{r}{125} = \frac{3}{5}.$$

Clear of fractions by multiplying both terms of the equation by 125.

Prove the correctness of your answer.

310. *To clear an equation of fractions, multiply both terms of the equation by the least common denominator of the fractions.*

11. $\frac{4}{5}$ is what per cent of $\frac{3}{5}$?

12. $\frac{3}{4}$ is what per cent of $\frac{4}{5}$?

$$\frac{4}{5} \times \frac{r}{100} = \frac{3}{4}; \quad i.e. \ \frac{r}{125} = \frac{3}{4}.$$

13. $3\frac{1}{3}$ is what per cent of $\frac{2}{3}$?

14. What per cent of $389.50 is $124.64?

15. $174.04 is 95% of what sum of money?

16. $\frac{3}{4}\%$ of a number is 81. What is the number?

$$\tfrac{3}{4}\tfrac{0}{0} \text{ of } b = 81.$$

17. 984 is $133\frac{1}{3}\%$ of what number?

18. What number increased by $33\frac{1}{3}\%$ of itself equals 984?

Let n represent the number.

Then $$n + \frac{n}{3} = 984; \ i.e. \frac{4n}{3} = 984.$$

Clearing of fractions, $4n = 984 \times 3 = 2952$. $n = 738$, *Ans.*

PROOF. — $738 + 33\frac{1}{3}\%$ of $738 = 738 + 246 = 984$.

19. What number increased by 25% of itself equals 85?

311. Oral Exercises.

1. 3 is what part of 6?
2. 3 is what decimal of 6?
3. 3 is how many hundredths of 6?
4. 3 is what per cent of 6?
5. 6 is what per cent of 3?
6. What number is 50% of 6?
7. 3 is 50% of what number?
8. 2 is what % of 100?
9. 2 is what % of 200?
10. What number is 5% of 100?
11. What % of 20 is 1?
12. 4 is what % of 200?
13. 3 is $\frac{1}{2}$% of what number?
14. What per cent of 9 is $20\frac{1}{4}$?

$\frac{9\,b}{100} = 20\frac{1}{4}$; $9\,b = 20\frac{1}{4}$ hundred; $b = 2\frac{1}{4}$ hundred $= 225$, *Ans.*

15. What number, increased by $\frac{1}{4}$ of itself, equals 10?
16. What number, increased by 25% of itself, equals 20?
17. 65 diminished by 5% of itself equals what?
18. Buying price \$ 100, selling price \$ 112.50. Gain %?
19. Cost \$ 80, profit 20%. Selling price?
20. What principal will give \$ 30 yearly interest at 6%?
21. A man had \$ 600 in the bank. He drew out $16\frac{2}{3}$ per cent of it. How many dollars remained in the bank?
22. A lost 40 per cent of his money, and had \$ 750 left. How much had he at first?

23. If I am compelled to lose $12\frac{1}{2}\%$ on damaged goods, how must I sell those that cost me $\$5.60$?

24. A man put $\$15$, which was $16\frac{2}{3}\%$ of his month's salary, in the bank. What was his month's salary?

25. If each boy eats 60% of a loaf of bread, how many boys will eat 6 loaves?

NOTE. — In the solution of the foregoing oral problems, pupils should not be compelled to use the method suggested for the written exercises.

REVIEW.

312. Written Problems.

1. A man receives from a bank 4% a year as interest on money he has in the bank. If his interest for a year is $\$60$, how much money has he in the bank?

2. A city had a population of 4500 at the end of 1903. The population at the end of 1904 was 1080 greater. What per cent did the population increase during the year?

$$1080 = \text{what per cent of } 4500?$$

3. A person who sold an article for 25% more than its cost, received $\$85$ for it. What was the cost?

$$\text{Cost} + \tfrac{1}{4} \text{ cost} = \$85.$$

4. A person receives $\$45$ annual interest on $\$1000$. What rate per cent does he receive?

5. A farmer sold $16\frac{2}{3}$ per cent of his sheep, and had 75 remaining. How many had he at first?

6. A clerk has an income of $\$1100$ per annum. He pays 20 per cent of it for board, $1\frac{1}{2}$ per cent for washing, 2 per cent for incidentals, 15 per cent for clothing, 9 per cent for other expenses, and loses in various ways 50 per cent of the amount then remaining. What sum does he have left?

7. What per cent of a school is boys, and what per cent girls, there being 640 of the former and 560 of the latter?

8. What per cent of 9.075 is 24.2?

9. How large a sale must a merchant make, at a profit of 15%, that his gain may be $3750?

10. A coal dealer bought 25,784 tons of coal at $5 a ton He sold 40% of it at $7, 20% of it at $8.50, and the remainder at $4.50. How much did he gain?

11. A man shipped 600 barrels of flour, and lost $16\frac{2}{3}\%$ of it by storm; he sold 75% of the remainder. What per cent of the whole remained?

12. $66\frac{2}{3}\%$ of 200 bushels is $2\frac{1}{2}\%$ of how many bushels?

13. If corn selling for 21¢ a bushel more than cost gives a profit of 30%, what did it cost?

14. $\frac{1}{3} + \frac{4}{9}$ of a number is what per cent of it?

15. A boy deposited $15 in bank. This was 30 per cent of what he had in bank before making this deposit. What had he there after this deposit?

16. A man can do a certain work in $18\frac{3}{4}$ days. What per cent of it can he do in $6\frac{2}{3}$ days?

17. A man spent 30 per cent of his money for clothes, 20 per cent for rent, and had $75 left. What rent did he pay?

18. What is the difference between $\frac{1}{2}$ per cent of $15,000 and 50 per cent of $15,000?

19. A pole extended into the mud $5\frac{5}{6}$ feet; $33\frac{1}{3}\%$ of its length was in the river and 25% of it in the air. What was the length of the pole?

20. There were 984 patients in a certain hospital, classified as follows: 369, pulmonary diseases; 246, nervous diseases; 123, diseases of heart; and 246, various other diseases. Give the per cent of each class.

APPLICATIONS OF PERCENTAGE.

313. Commission. — The term *per cent* occurs in many business transactions. A person who sells goods for another receives a certain per cent of the amount he obtains for the goods, as a *commission.* A person who buys goods for another is paid a *commission,* which is a certain per cent of the cost of the goods. A person who collects money for another is paid a *commission* of a certain per cent of the amount collected.

Commission is a percentage paid to an agent for his services.

314. Insurance. — The owner of property who desires to be insured for a definite sum pays some per cent of this sum for the insurance. The amount he pays is called the *premium.* The document given by the insurance company as a receipt is called a *policy.* It states the agreement of the company to pay the owner of the property a sum equivalent to the loss sustained, provided that it does not exceed the sum for which the owner is insured. Thus, the owner of a house valued at $5000 may insure it against fire for $4000. If the house is injured to the extent of $4000 or less, the owner receives from the company the amount of the loss.

Insurance is a contract by which one party agrees to pay to another a specified sum in case of loss or damage.

NOTE. — The teacher should show pupils an insurance policy, and read the contract made by the company as expressed therein.

315. Duties. — The United States government collects from the importers of certain classes of goods a stated percentage of the value of the goods. This charge is called a *duty.*

Duties are taxes on imported goods.

NOTE. — Some duties are based upon a certain rate per square yard, per pound, etc.

316. Taxes. — For the expenses of maintaining a city, property owners pay a certain percentage of the valuation of their property as determined by the proper officials. The money thus collected from the owner is called a *tax*. The value fixed by the authorities is called the *assessed value*, which is generally somewhat less than the actual value.

A tax is a sum of money levied on persons or property for public purposes.

NOTE. — In many places the tax rate is fixed at so many thousandths of the assessed value.

The following ten oral and twenty written problems involve no new principles. The general formula $b \times \frac{r}{100} = p$ is applicable to each of them. The accompanying statement shows the *base* on which the percentage is calculated in certain classes of examples; also the name given to the *percentage*.

	Base	*Percentage*
COMMISSION . .	Value of goods bought or sold, etc.	Commission Brokerage
INSURANCE . .	Sum for which property is insured	Premium
TAXES	Assessed value of property	Taxes
DUTIES . . .	Value of goods imported	Duty

317. Oral Problems.

1. An agent collected a bill, and sent to his employer the amount, less $2\frac{1}{2}\%$ commission. If his commission was \$ 1.60, how much did he remit to his employer?

2. My house, worth \$ 12,000, is insured for $\frac{3}{4}$ of its value, at $\frac{1}{4}\%$. What premium do I pay?

3. A man collected a bill of $300 for me, at $\frac{1}{2}\%$ commission. How much was his commission?

4. Mr. Eastman collects bills for me, and I pay him $12\frac{1}{2}\%$. He pays over to me $56. How much did he collect?

5. What is the premium for insuring $3600 on my house at $\frac{3}{4}\%$?

6. What will it cost to insure a house worth $5000, at $\frac{1}{2}\%$ premium?

7. Find the duty at 35% on goods valued at $2000.

8. My taxes for 1904 are $175. The rate is $1\frac{3}{4}$ per cent. What is the assessed value of my property?

9. My agent collects the yearly rent of my house, and retains $15, the amount of his commission at $2\frac{1}{2}$ per cent. For how much does the house rent per year?

318. Written Problems.

1. How much insurance does a man receive for $12.50 when the rate is $2\frac{1}{2}\%$?

2. An importer paid duties amounting to $386.75. If the duty was 25% of the cost of the goods, what was their cost?

3. A collector deducts $2\frac{1}{2}\%$ commission, and returns to his employer $745.68. How much did he collect?

Let x represent the sum collected. Then $2\frac{1}{2}\%$ of x, or $\frac{x}{40}$, will represent the commission; and $x - \frac{x}{40}$, or $\frac{39x}{40}$, will represent the amount returned to the employer.

$$\frac{39x}{40} = 745.68.$$

Clearing of fractions: $39x = 29{,}827.20.$

$$x = 764.80. \quad \textit{Ans.}\ \$764.80.$$

It will be noted that only abstract numbers are used in an equation, the denomination being supplied in the answer.

4. The tax rate of a certain city is $1\frac{3}{4}\%$ upon the assessed value of property. If this value is 75% of the actual value, how much taxes does Mr. Smith pay upon a house and lot, the actual value of which is $\$ 24{,}000$?

5. The tax on an assessment of $\$ 8500$ is $\$ 48.45$. Required the rate on $\$ 1000$ of assessment.

6. Find the amount of an agent's sales, when his commission at 5 per cent amounts to $\$ 37.65$.

7. An agent buying wheat is offered a commission of $4\not{c}$ per bushel, or one of $4\frac{1}{2}$ per cent, and he chooses the former. The average price paid per bushel is $91\frac{1}{2}\not{c}$. Does he gain or lose by his choice, and how much per bushel?

8. A commission of $\$ 121.29$ was charged for selling $\$ 1866$ worth of goods. What was the rate of commission?

9. A man insured his house for $\$ 6500$, his store for $\$ 3500$, and his goods for $\$ 7000$, at $\frac{1}{2}\%$. What did his insurance come to?

10. If a piece of property is taxed $\$ 28.60$, at a tax rate of $\frac{4}{5}$ of one per cent, what is the assessed value of the property?

11. A house valued at $\$ 24{,}000$ was insured for two-thirds of its value, at $\frac{3}{5}\%$. What is the premium?

12. An agent collected 20% of an account of $\$ 750$, charging 4% commission. What was his commission, and what sum should he have paid over?

13. Paid $\$ 27$ for an insurance policy on my house. If the rate is $\frac{3}{4}\%$, for how much is my house insured?

14. My agent collected 80 per cent of a debt of $\$ 4500$, and charged $7\frac{1}{2}$ per cent commission. What amount should he pay me?

15. A farmer bought 6 cows through an agent. He sent $525.30 to pay for the cows and a commission of 3%. How much did each cow cost?

16. What will be a broker's commission, at $2\frac{7}{8}\%$, for selling a farm of 673 acres @ $52 per acre?

17. If the tax rate is $13.80 on $1000, what is the assessed value of property that pays a tax of $144.90?

18. A house is insured for $\frac{2}{3}$ of its value at $\frac{7}{8}\%$. The annual cost (premium) is $8.40. What is the value of the house?

Let x represent the value. Then $\frac{2x}{3} \times \frac{7}{800}$, or $\frac{7x}{1200}$, will represent the premium.

The equation becomes $\frac{7x}{1200} = 8.40$.

19. What will be the taxes on a house worth $48,000 and assessed at $\frac{2}{3}$ of its value, the tax rate being $18.50 per $1000 of assessed value?

20. A commission merchant receives $2\frac{1}{2}\%$ commission for buying grain for a customer. The cost of the grain and his commission amount to $4223. How much does the grain cost?

Let x represent the cost of the grain; $\frac{x}{40}$ will be the commission.

21. An importer paid $134.40 duties on imported goods valued at $384. Find the rate.

22. What is the duty in United States money on glass ware valued at 1500 francs, the rate being 60%, and the franc being worth 19.3 cents?

23. Find the duty on a gross of scissors, valued at $2.50 per dozen, the rate being 75 cents per dozen and 25% on the value.

PROFIT AND LOSS.

In determining the rate per cent of gain or loss on goods sold, the buying price of the goods is taken as the base.

319. Oral Problems.

1. What is the gain per cent on sugar bought at 5 cents per pound and sold at 6 cents per pound?

Profit 1¢, which is one-fifth of buying price, or 20%.

2. By selling a house for $3500, I lose $500. What is my loss per cent?

The loss, $500, is what per cent of the cost, $4000?

3. By selling a lot for $1000, Mr. Jones loses 20 per cent. What did the lot cost?

The selling price, $1000, is four-fifths of the cost.

4. Find the cost of an article which was sold for $60, at a loss of 70%.

5. If I buy a dozen pencils at 2¢ each, and sell at 3¢ each, what is the gain per cent?

6. A saddle was sold for $18, which was $12\frac{1}{2}\%$ more than the cost. How much did it cost?

7. What % is gained on goods sold at double the cost?

8. Sold flour at a profit of $2, and gained 25%. What was the cost per barrel?

9. What is the % of gain, when boots which cost $2 a pair are sold for $2.50?

10. What per cent is lost in buying potatoes at 80¢ a bushel, and selling them at 60¢ a bushel?

11. If I buy butter at 30¢ a pound, how much per cent do I gain by selling it at 36¢ a pound?

320. Written Exercises.

Find the profit or the loss, and the selling price:

1. Cost \$1876; gain 15%.

Gain = 15 per cent of \$1876. Selling price = cost + gain.

2. Cost \$36.75; loss 20%.
3. Cost \$1012.50; gain $16\frac{2}{3}$%.
4. Cost \$875; loss 5%.
5. Cost \$934.56; gain $12\frac{1}{2}$%.

Find the profit or the loss per cent.

6. Cost \$600; selling price \$618.

\$18 = ? % of \$600.

7. Cost \$1203; selling price \$802.
8. Cost \$86.20; selling price \$73.27.
9. Cost \$908.40; selling price \$1090.08.
10. Cost \$84; selling price \$78.75.
11. Selling price \$78.75; loss \$5.25.

NOTE. — Cost = \$78.75 + \$5.25 = \$84.

12. Selling price \$150; gain \$25.

Use the *cost* (\$150 − \$25) as the base.

13. Selling price \$831.25; loss \$43.75.
14. Selling price \$1051.38; gain \$116.82.
15. Selling price \$843.75; loss \$168.75.

Find the cost, and the profit or loss:

16. Selling price \$468.75; gain 25%.

Representing the cost by x, the selling price is $\frac{5x}{4}$.

$$\frac{5x}{4} = 468.75.$$

17. Selling price $73.84; loss 20%.

18. Selling price $1646.08; gain 33⅓%.

19. Selling price $204; loss 15%.

20. Selling price $66.30; gain 4%.

21. A man buys a horse for $275, and sells it at a profit of 20 per cent. How much does he gain?

22. A cow is sold for $75, on which the profit is $15. What is the gain per cent?

23. A lot is sold for $960, which is 20 per cent more than it cost. Find the cost of the lot.

24. Tea that costs 32¢ per pound is sold for 48¢. What is the gain per cent?

25. A man buys a horse for $175 and sells it for $200. What per cent does he gain?

26. What per cent was lost on a horse that cost $200, and that was sold at a loss of $25?

27. What is the selling price of dress goods costing 33⅓¢ per yard, on which a profit of 12½ per cent is made?

28. Sold a coat for $33.60, thereby losing 16 per cent. What was its cost?

29. How much did I gain on a house for which I paid $8760, my profit being 2½ per cent?

30. A man paid for a house $4500, and for repairs $150, and then sold it for 18% above the entire cost. What did he receive for it?

31. To make 15 per cent profit, what must goods be marked that cost 96 cents per yard?

32. Goods costing 96 cents per yard are marked at 25% advance, what per cent is gained if they are sold 10% below the marked price?

33. Find the per cent of profit on apples bought at $1.25 per bushel, and sold at 25 cents per half peck.

COMMERCIAL DISCOUNT.

321. Wholesale dealers in certain classes of goods allow to purchasers of large quantities a deduction from the prices printed in their catalogues. This is called a *trade discount* or *commercial discount.* A discount for prompt payment is also frequently allowed. The following bill contains a trade discount and a discount for cash:

PHILADELPHIA, *Jan. 17, 1904.*

THE OCEAN BATHING SUIT CO.

Terms: Cash less 5 per cent. *Sold to* MR. J. H. HAAREN.

		$12\frac{6}{12}$ *doz. Suits*	*$18*	*$225*	—	
			less 15%	*33*	*75*	
				$191	*25*	
			Cash less 5%	*9*	*56*	
						$181 *69*
		Rec'd Paym't, Jan. 17, 1904, O. B. S. CO. per M. M.				

1. Make out a bill for 16 gross of roman candles at $26.75 per gross, less 60%.

2. On a bill of goods amounting to $583.40, a discount of 5% is given for cash. What is the amount paid?

3. Sept. 1, 1905, Mr. Maxwell bought tea amounting to $1876.50. If 5% is deducted for payment within ten days, how much should he pay if he paid the bill Sept. 9?

4. What will be the cost of 15 cases cocoa @ $13.20 each, less 20%?

5. Bought 5 gross of essence of lemon at 50¢ per doz., less 5%. What is the amount of my bill?

6. Find the cost of 15 cases of chloride of lime, 50 lb. per case, at $9\frac{1}{2}$¢ per pound, less 15%.

7. Find the cost of a wagon, the catalogue price of which is $750, the discount being 30%.

8. What will be the cost of goods amounting to $1837.60, on which there is allowed a discount of $17\frac{1}{2}$%?

9. Find the net cost of 1630 yd. silk, invoiced at $1.10 per yard, less 16% discount.

NOTE. — The amount previous to the deduction of the discount is known as the *gross* amount. The *net* amount or the net cost is the sum actually due after the deduction of the discount.

10. What is the cost, in francs, of 843.72 meters silk, at 5.75 francs per meter, less 12%?

11. What will be the net cost of a bill of plated ware amounting to $84.75, on which a discount of $33\frac{1}{3}$ and 10% is allowed?

	$84.75
less $\frac{1}{3}$	28.25
	56.50
less $\frac{1}{10}$	5.65
Ans. $	net.

When more than one discount is given, each successive discount is based on the remainder left after the deduction of the previous discount.

NOTE. — The mark % is generally written only after the last rate.

12. Find the difference between $390 less $43\frac{1}{3}$% discount, and $390 less $33\frac{1}{3}$ and 10% discount.

13. An army fought two battles. In the first it lost 15 per cent, and in the second 20 per cent of the original number, after which it mustered 19,500 men. What was the original strength of the army?

14. Find the net cost of 18,500 bags at $4.40 per M, less 60 and 10 and 5%.

15. What is the net cost of a lot of musical instruments amounting to $1875.60, on which a discount of 10, 5, and $2\frac{1}{2}$% is allowed?

16. What would be the net cost of the same articles, if the discount were $2\frac{1}{2}$, 5, and 10%?

17. Find the net cost of the same, at $17\frac{1}{2}$% discount.

18. Which is the better discount for the buyer, 40 and 10% or 30 and 20? What will be the difference on a bill of $100?

19. $100 less $33\frac{1}{3}$ and 10% discount is equal to what? What per cent discount is $33\frac{1}{3}$ and 10% equal to?

20. $100 less 10 and $33\frac{1}{3}$% is equal to what?

NOTE. — The pupil will note that the result is the same as in Problem 19.

21. A man marks an article $1.50, and sells it at a discount of 25% from the marked price. If the article cost him 90¢, what is his gain per cent?

22. John Jasper & Co. sold the following goods. Make out the bill, less 50 and 10 and 10 and 10% discount.

500	$\frac{1}{4}$-pound bags at	$1.00	per M.
1500	$\frac{1}{2}$-pound bags at	1.20	per M.
3000	1-pound bags at	1.60	per M.
5500	$1\frac{1}{2}$-pound bags at	1.70	per M.
2000	2-pound bags at	2.00	per M.

NOTE. — In making out large numbers of bills clerks have no time for unnecessary words. The first item would be written as follows:

500 $\frac{1}{4}$ lb. Bags $1. .50

the words "at" and "per M" being considered unnecessary.

322. Oral Problems.

1. A piano, marked \$ 800, is sold at a discount of 25 and 10%. What is the selling price?

2. Bought goods amounting to \$ 600, less 5% for cash. What is the net cost of the goods?

3. What single discount is 50 and 10% equal to?

Taking \$ 100 as a base, 50 % discount deducts \$ 50 and leaves \$ 50. 10 % deducts \$ 5, leaving \$ 45. The total deduction is \$ 55; the single equivalent discount is 55 %.

4. What single discount is 30 and 30% equal to?

Representing the base by x, the first discount is 30 % of x, or $\frac{30\,x}{100}$, leaving $\frac{70\,x}{100}$. The second discount is $\frac{3}{10}$ of $\frac{70\,x}{100}$, which is $\frac{21\,x}{100}$. The two discounts are $\frac{30\,x}{100}$ and $\frac{21\,x}{100}$, which make a total of $\frac{51\,x}{100}$, or 51 % of the base.

5. Paid \$ 729 for goods, on which 10% was allowed. What was the "gross" price?

6. How much will be paid for 12 doz. bottles flavoring extract, at 60 ¢ per dozen, less 10% ?

7. What is the "list" price of an article for which I paid \$ 48, after a discount of 25% was deducted?

Note. — The "list" price, "gross" price, or "catalogue" price is the price before the deduction of discounts.

8. What is the net price of an article catalogued at \$ 880, on which there is a discount of 75% ?

Note. — A discount of 75 % from the list price means that the net price is 25 % of the list price. Instead, therefore, of finding 75 % of \$ 880 and deducting it from \$ 880, the pupil should shorten the work by taking 25% of \$ 880.

9. 75 is 25% more than what number?

10. Find the cost of a wagon "catalogued" at \$ 700, the discount being 30%.

Note. — The cost is 70 % of \$ 700.

INTEREST.

323. Preliminary Exercises.

1. A farmer, needing $1000 to purchase additional land, borrows the money, agreeing to repay it at a given time with 6% of the sum for each year he has the use of it. This annual payment of $60 for the use of $1000 is called *interest*. The $1000 is called the *principal*. If the borrower repays the $1000 at the end of two years and also $120 interest, the total payment of $1120 is called the *amount*.

2. What is the interest on $1000 at 6% for 6 months?

3. What is the amount of $1000 for 3 years at 6%?

4. What is the interest on $1000 for 1 month at 6%?

5. Taking 30 days to a month, find the interest on $1000 for 15 days at 6%.

324. Written Exercises.

1. Find the interest on $750 at 6% for 2 years 6 months.

The interest for 1 year is found by multiplying the principal, $750, by the rate, 6 hundredths, and this product by the number of years, $2\frac{1}{2}$.

$750 Principal.
× .06 Rate.
Interest for 1 yr. $45.00
× $2\frac{1}{2}$ Time.
$90.
22.50
Interest for 2 yr. 6 mo. $112.50

The foregoing may be expressed by the formula:

$$\text{Principal} \times \frac{\text{Rate}}{100} \times \text{Time (in years)} = \text{Interest}$$

It is suggested that the work be arranged in this manner, so that it may be shortened by cancellation.

$$\$750 \times \frac{6}{100} \times \frac{5}{2} = \frac{\$225}{2} = \$112.50, \textit{ Ans.}$$

2. What is the interest on \$84.75 at 4% for 3 months 6 days?

3 months 6 days = 96 days = $\frac{96}{360}$ year.

The 100 in the divisor is cancelled by removing the decimal point in the principal two places to the left.

$$\$.84\tfrac{75}{100} \times \frac{4}{100} \times \frac{\overset{16}{\cancel{96}}}{\underset{90}{\cancel{360}}} = \$.9040$$

Ans. 90 cents. 6

3. Find the interest on \$394.50 for 2 years 7 months 24 days at $4\frac{1}{2}$%.

$$\frac{\overset{477}{\$.394\tfrac{50}{100} \times \cancel{9} \times \cancel{954}}}{\underset{4}{\cancel{200} \times \cancel{360}}} = \frac{\$188.1765}{4} = \$47.041+. \quad \textit{Ans.}\ \$47.04.$$

The time is readily changed to days: 720 + 210 + 24 = 954.

This is expressed in years by placing 360 in the divisor, *i.e.* below the line. The three ciphers in the divisor are cancelled by moving the decimal point in the principal three places to the left.

In calculating interest, take 30 days to a month, 12 months to a year.

Find the interest on

4. \$308 at 5% for 20 days.
5. \$360 at 5% for 33 days.
6. \$720 at 7% for 21 days.
7. \$1000 at 5% for 8 days.
8. \$94.43 at 7% for 2 mo. 3 da.
9. \$464.75 at 6% for 8 mo. 12 da.
10. \$400 at $4\frac{1}{2}$% for 1 yr. 1 mo. 1 da.

325. *Amount = Principal + Interest.*

Find the amount:

1. \$813, from April 19, 1902, to March 4, 1907, at 6%.

The time is found by compound subtraction.

1907	3	4
1902	4	19
4	10	15

4 yr. 10 mo. 15 da.

$$\text{Interest} = \frac{\overset{.271}{\$.\cancel{813}\tfrac{}{}} \times \cancel{6} \times 1755}{\underset{2}{\cancel{100} \times \cancel{360}}} = \$237.80 +$$

Amount = \$813 + \$237.80 = \$1050.80, *Ans.*

2. $960, from Jan. 1, 1903, to Dec. 21, 1904, at 4%.
3. $27.84, for 3 yr. 6 mo. 9 da., at 6%.
4. $48.90, for 17 da., at 6%.
5. $144, for 2 yr. 5 da., at $3\frac{3}{4}$%.
6. $834.76, for 15 mo. 27 da., at $4\frac{1}{2}$%.
7. $5760, for 1 yr. 5 mo. 29 da., at 5%.
8. $2346.50, for 7 yr. 13 da., at 3%.
9. $1892, for 3 yr. 5 mo., at 7%.
10. $150.40, for 1 yr. 2 mo. 3 da., at 6%.

326. Interest-bearing Demand Notes.

A promissory note is a written agreement to pay a stated sum of money after a given time or on demand to a certain person with or without interest. The person signing the note below, James Dunne, is called the *maker;* the person in whose favor it is drawn, Charles C. Wise, the *payee.* If the latter wishes to transfer it to James H. Tully, he writes on the back of the note: Pay to the order of James H. Tully; and underneath he signs his name. This is called an *indorsement in full.* By merely signing his name on the back, which is called an *indorsement in blank,* Mr. Wise makes it payable to any person holding it. The effect of an indorsement is also to make the indorser liable in the event of the maker refusing to pay.

1. San Francisco, Jan. 7, 1902.

On demand, I promise to pay Charles C. Wise, or order, Seven Hundred Sixty-five $\frac{40}{100}$ Dollars, value received, with interest at 6 per cent.

$\$765\frac{40}{100}$. James Dunne.

How much money will be required to pay the above note, with interest, July 15, 1903?

2. A demand note, dated Sept. 25, 1902, with interest at 8% from date, is paid Jan. 2, 1905. How much was due, the face of the note being $750?

3. Find the amount due March 4, 1904, on a note for $365.84, dated May 20, 1902, with interest from date at 7%.

4. Find the amount necessary, Oct. 16, 1906, to pay a note of $1240, with interest at 6% from Aug. 15, 1902.

5. An interest-bearing note for $87.60 is dated April 3, 1900. How much is due on it for principal and interest Jan. 2, 1908? Rate 4½%.

327. Oral Problems.

If these are first used as sight problems, an opportunity will be afforded to develop different methods for solving many of them.

1. Find the interest on $300, for 1 yr. 7 mo., at 4%.

$12 per year is how much for 7 months?

2. On $60, for 33 days, at 6%.

$3.60 for 360 days is how much for 33 days?

3. On $120, from Jan. 1, 1903, to July 1, 1904, at 5%.

4. How long will it take $100 to produce $15, interest at 6%?

5. At what rate per cent will $50 produce $6 in 2 years?

6. What is the interest on $300, at 6%, from Feb. 1 to Feb. 21?

7. What part of a year is 72 days?

8. Find the interest at 4%, for 90 days, on $150.

9. On $240, for 36 days, at 5%.

10. What is the amount of $200, for 3 yr. 1 mo., at 6%?

11. How long will it take $1 to make $1 interest at 5%?

12. How long will it take any sum to double itself at 6%?

13. How long will it take $14.90 to double itself at 4%?

PARTIAL PAYMENTS.

328. United States Rule.

When the maker of an interest-bearing note pays a portion of the debt represented by the note, the money is applied in the first place to the payment of the interest, then to the reduction of the principal.

1. DULUTH, MINN., Jan. 5, 1902.

On demand, I promise to pay to the order of James F. McGee Three Hundred Dollars, value received, with interest at 7 per cent.

300\frac{00}{100}$. J. RANDOLPH PAGE.

Payments: May 20, 1902, $100; Oct. 30, 1902, $100; March 6, 1903, $50.

How much was due Jan. 5, 1904?

Find amount of $300 Jan. 5, 1902, to first payment May 20, 1902, 4 mo. 15 da. (by compound subtraction),	$307.88
Deduct first payment,	100.00
Balance May 20, 1902,	$207.88
Interest on $207.88 to Oct. 30, 5 mo. 10 da.,	6.47
Amount,	$214.35
Less second payment,	100.00
Balance Oct. 30, 1902,	$114.35
Interest on $114.35 Oct. 30 to March 6, 4 mo. 6 da.,	2.80
Amount,	$117.15
Less third payment,	50.00
Balance March 6, 1903,	$67.15
Interest on $67.15 March 6 to Jan. 5, 9 mo. 29 da.,	3.90
Due Jan. 5, 1904,	$71.05

Find the amount of the principal to the time when the payment or the sum of two or more payments equals or exceeds the interest.

From this amount deduct the payment or sum of payments. Use the balance then due as a new principal, and proceed as before.

2. How much is due June 3, 1905, on a demand note for $1200, with interest at 6 %, dated June 3, 1902, bearing indorsements of payment of $500, Sept. 18, 1903; $600, Jan. 3, 1904?

NOTE. — Anything written on the back of a document is called an *indorsement.* Payments made are usually written on the back of the notes.

3. A demand note for $600, bearing interest at 5 %, was given Feb. 18, 1902. A payment of $250 was made May 28, 1903; one of $150 was made Oct. 8, 1903. How much is due Jan. 23, 1905?

4. A note for $2000, with interest at 7 %, was dated April 15, 1901. Indorsements were made as follows: $50, Sept. 20, 1901; $100, May 26, 1902; $1000, June 20, 1903. How much is due Dec. 27, 1904?

Face of note,	$2000.00
Interest from April 15 to Sept. 20, 1901, 5 mo. 5 da.,	60.28
Amount due Sept. 20, 1901,	$2060.28
If the $50 payment were deducted, and interest computed on the balance, $2010.27, the maker would be charged interest on $10.27 more than the face of the note, and this the law does not allow. Interest is taken on $2000 until next payment, May 26, 1902, 8 mo. 6 da.,	95.67
Amount due May 26, 1902,	$2155.95
As the two payments are not large enough to meet the interest now due, the interest is again computed on the original $2000 from May 26, 1902, to June 20, 1903, 1 yr. 24 da.,	149.33
Amount of $2000 from April 15, 1901 to June 20, 1903,	$2305.28
Less $50 + $100 + $1000 (three payments),	1150.00
Balance due June 20, 1903,	$1155.28
Interest on $1155.26 to Dec. 27, 1904, 1 yr. 6 mo. 7 da.,	122.87
Due Dec. 27, 1904,	$1278.15

5. ALBANY, N.Y., March 5, 1903.

One year after date, I promise to pay John Harrigan, or order, Nine Hundred Dollars, value received, with interest at six per cent.

900\frac{00}{100}$. ANDREW T. SULLIVAN.

Indorsed as follows: June 5, 1903, $10; Sept. 5, 1903, $50; Jan. 5, 1904, $120. What was due March 8, 1904?

6. ALEXANDRIA, LA., June 19, 1903.

On demand I promise to pay to the order of George H. Dotzert, Two Thousand Four Hundred Fifty-four $\frac{75}{100}$ Dollars, value received, with interest at 6 per cent.

2454\frac{75}{100}$. CHARLES W. LYON.

The following payments were made: July 5, 1903, $450; Sept. 18, 1903, $700; Oct. 25, 1903, $300. Find the amount due Jan. 2, 1904.

329. In the United States courts, and in those of some of the states, interest for a portion of a year is taken by days, upon the basis of 365 days to the year. To make the work easier for the pupils, however, the year of 360 days should be used in the examples given, and the time between dates should be found by compound subtraction.

330. Merchants' Rule.

The merchants' rule is frequently used where all the payments are made within a year.

The interest is computed on the face of an interest-bearing note from its date until settlement, and interest is allowed on all credits from their payment until settlement.

The exact number of days is taken, and the interest is computed on the basis of 360 days to the year.

BOSTON, MASS., June 19, 1903.

On demand, I promise to pay Charles R. Buttrick, or order, Two Thousand Four Hundred Fifty-four $\frac{75}{100}$ Dollars, value received, with interest at 6 per cent.

2454\frac{75}{100}$. JOHN J. P. FAGAN.

The following payments are endorsed on the note: July 5, 1903, $200; July 29, 1903, $450; Sept. 18, 1903, $700; Oct. 25, 1903, $300.

Find the amount due Jan. 2, 1904.

If no payments had been made, there would be due,		$2454.75
And interest from June 19 to Jan. 2, 197 days,		80.60
Total due,		$2535.35
The credits are: Payment July 5, 1903,	$200.00	
Interest on $200, July 5 to Jan. 2, 181 days,	6.03	
Payment July 29, 1903,	450.00	
Interest on $450, July 29 to Jan. 2, 157 days,	11.78	
Payment Sept. 18, 1903,	700.00	
Interest on $700, Sept. 18 to Jan. 2, 106 days,	12.37	
Payment Oct. 25, 1903,	300.00	
Interest on $300, Oct. 25 to Jan. 2, 69 days.	3.45	1683.63
Balance due,		$851.72

Find the amount of an interest-bearing note at the time of settlement.

Find the amount of each credit from its time of payment to the time of settlement; subtract their sum from the amount of the note.

331. Written Exercises.

1. A note for $500, with interest at 6 %, is dated July 25, 1904. Payments are made: $100, Sept. 18; $200, Feb. 5, 1905. How much is due April 1, 1905?

2. Find amount due Sept. 15, 1903, on a demand note for $1875, with interest at 6 %, dated Jan. 18, 1903. Payments of $1000 and $500 were made March 30 and June 17, respectively.

3. June 12, 1904, Robert Colgate bought goods amounting to $600. Dec. 31, 1904, he paid $300; April 5, 1905, $200; June 1, 1905, he settled the account. How much did he pay on that date, if he is charged 6 % on the purchase from its date, and is allowed 6 % interest on his payments?

4. John C. Kelley loaned Chas. R. Robertson $500, Sept 1, at 6%. Payments of $200 each were made Oct. 1 and Nov. 1. How much is due Dec. 1?

Dr. HORACE E. DRESSER *Cr.*

1905.					1905.				
Feb.	5	To merchandise,	840	00	Mar.	9	By cash,	500	00
Dec.	31	To interest to date,			Sept.	13	By cash,	200	00
					Dec.	31	By interest to date,		
					"	"	By cash,		

5. Find the amount paid in settlement of the foregoing account, Dec. 31, 1905. Interest 6%.

6. A merchant's books show the following debits: Feb. 13, merchandise, $725.00; April 14, merchandise, $603.00. The credits are: April 5, cash, $600; Aug. 29, cash, $300. How much is due Oct. 5, interest 6%?

332. Oral Exercises.

1. If I sell for $4.50 a book which cost me $3, what per cent do I gain?

2. What is the interest of $200, for 90 days, at 3%?

3. One acre of corn yields 80 bushels, and another acre 20% more. What does the second acre yield?

4. What will it cost to fence a garden 10 rods long and 6 rods wide, at $1 a rod?

5. In a certain school 40 pupils are present and 10 are absent. What per cent are absent?

6. What is the difference between a floor 40 feet square and two others each 20 feet square?

7. What is the interest of $12, for 1 yr. 4 mo., at 6%?

8. If $2\frac{1}{2}$ pecks of berries cost one dollar, what would 3 quarts cost at the same rate?

9. Bought 5 bushels nuts at a dollar a peck, and got 5% off for cash. How much did I pay for the nuts?

333. Written Problems.

1. Gold coin contains 90 per cent gold, 9 per cent silver, 1 per cent copper. Find the quantity of each metal in 50 double-eagles ($20), each containing 516 grains.

2. A, B, and C buy a farm. A pays $8700, B pays $7200, C pays $4100. What per cent of the purchase money does each furnish?

3. The one-cent pieces weigh 48 grains. How many dollars would weigh 120 pounds avoirdupois (7000 grains to pound)?

4. If a person lends me $250 for 8 months, for how long ought I to lend him $400 as an equivalent?

5. Goods costing $8 are sold at an advance of 20 per cent. The marked price is $12. What per cent reduction is made on the marked price?

6. There are 5 boys whose heights are 4 ft. 9 in., 5 ft. 1 in., 4 ft. 5 in., 3 ft. 11 in., and 4 ft. 4 in., respectively. What is their average height?

7. In the written number 185.4, the number expressed by the first two (left-hand) figures is how many time the value expressed by the second two figures?

8. Express decimally, and also as a common fraction, the value of each of the following: 115 per cent; $\frac{1}{16}$ of 1 per cent; $\frac{13}{32}$ of 1 per cent.

9. M bought $\frac{5}{27}$ of a manufacturing business for $3517.85, and N bought $\frac{4}{19}$ of the same business at the same rate. How much did N's interest cost him?

334. To find Principal, Rate, or Time.

1. What principal will produce $2.88 interest in 8 months at $4\frac{1}{2}\%$?

The interest on $1 at $4\frac{1}{2}\%$ for 8 months is $1 $\times \frac{9}{200} \times \frac{8}{12}$, or $.03. Since $.03 is produced by $1 (at the given rate for the given time), $2.88 will require a principal of as many dollars as $.03 is contained times in $2.88, or 96. *Ans.* $96.

PROOF. $96 $\times \frac{9}{200} \times \frac{8}{12} = \$2.88$.

2. What principal will amount to $98.88 in 8 months at $4\frac{1}{2}\%$?

The amount of $1 at $4\frac{1}{2}\%$ for 8 months is $1.03. If an amount of $1.03 is produced from a principal of $1, an amount of $98.88 will be produced from a principal of as many dollars as $1.03 is contained times in $98.88, or 96. *Ans.* $96.

To find the **principal,** *divide the given interest (or amount) by the interest (or amount) of* **$1** *at the given rate for the given time.*

The following is an algebraic method of solving No. 1:

(1) Let x represent the required principal.

(2) The interest will be $x \times \frac{9}{200} \times \frac{8}{12}$, or $\frac{3x}{100}$.

(3) $\frac{3x}{100} = 2.88$.

(4) Clearing of fractions, $3x = 288$.

(5) Dividing, $x = 96$. *Ans.* $96.

The following is an algebraic method of solving No. 2:

(1) Let x represent the required principal.

(2) The interest will be $x \times \frac{9}{200} \times \frac{8}{12}$, or $\frac{3x}{100}$.

(3) The amount will be $x + \frac{3x}{100}$, or $\frac{103x}{100}$.

(4) $\frac{103x}{100} = 98.88$.

(5) Clearing of fractions, $103x = 9888$.

(6) Dividing, $x = 96$. *Ans.* $96

3. At what rate per cent will $723.60 produce $36.18 interest in 1 yr. 1 mo. 10 da.?

The interest on $723.60 at 1% for 1 yr. 1 mo. 10 da. is $723.60 $\times \frac{1}{100} \times \frac{400}{360}$, or $8.04. Since $8.04 is produced by a rate of 1%, $36.18 will require a rate of as many per cent as $8.04 is contained times in $36.18, or $4\frac{1}{2}$. *Ans.* $4\frac{1}{2}$%.

PROOF. $\$723.60 \times \frac{9}{200} \times \frac{400}{360} = \$36.18.$

4. At what rate per cent will $723.60 amount to $759.78 in 1 yr. 1 mo. 10 da.?

Find the interest by subtracting the principal $723.60 from the amount $759.78, and proceed as in No. 3.

To find the **rate**, *divide the given interest by the interest at* **1**% *on the given principal for the given time.*

The following is an algebraic solution of Nos. 3 and 4:

(1) Let x represent the rate.

(2) The interest will be $723.60 \times \frac{x}{100} \times \frac{400}{360}$, or $8.04\,x$.

(3) $8.04\,x = 36.18.$

(4) Clearing of decimals, $804\,x = 3618.$

(5) Dividing, $x = 4\frac{1}{2}$. *Ans.* $4\frac{1}{2}$%.

5. In what time will $85.50 produce $8.17 interest at 4%?

The interest on $85.50 at 4% for 1 year is $85.50 $\times \frac{4}{100}$, or $3.42. Since $3.42 is produced in 1 year, $8.17 will require as many years as $3.42 is contained times in $8.17, or $2\frac{7}{18}$.

Ans. $2\frac{7}{18}$ years, or 2 yr. 4 mo. 20 da.

PROOF. $\$85.50 \times \frac{4}{100} \times 2\frac{7}{18}$
$= \$85.50 \times \frac{4}{100} \times \frac{43}{18} = \$8.17.$

6. In what time will $85.50 amount to $93.67 at 4%?

Find the interest by subtracting the principal $85.50 from the amount $93.67, and proceed as in No. 5.

To find the **time**, *divide the given interest by the interest for* **1 year** *on the given principal at the given rate.*

335. The following is an algebraic solution of Nos. 5 and 6:

(1) Let x represent the time in *years*.

(2) The interest will be $85.50 \times \frac{4}{100} \times x$, or $3.42\,x$.

(3) $3.42\,x = 8.17$.

(4) Clearing of decimals, $342\,x = 817$.

(5) Dividing, $x = 2\frac{7}{18}$.

Ans. $2\frac{7}{18}$ years, or 2 yr. 4 mo. 20 da.

336. The algebraic method consists (1) in representing the unknown quantity (principal, rate, or time) by x; (2) finding the interest, by multiplying principal by rate by time; (3) forming an equation, by making this product equal to given interest; (4) solving the equation.

337. Written Exercises.

Find rate, time, etc.

1. Principal, \$ 2000; time, 3 yr.; interest, \$ 300. Rate?

2. Principal, \$ 1800; rate, 4%; interest, \$ 144. Time?

3. Time, 8 mo.; rate, $4\frac{1}{2}$%; interest, \$ 2.88. Principal?

4. Principal, \$ 38; time, 2 yr.; amount, \$ 40.28. Rate?

5. Principal, \$ 140; rate, $3\frac{1}{2}$%; time, 3 mo. 15 da. Interest?

6. Amount, \$ 39.60; rate, 4%; time, 2 yr. 6 mo. Principal?

7. Amount, \$ 484.15; rate, $3\frac{1}{2}$%; principal, \$ 460. Time?

8. Principal, \$ 39.60; rate, 4%; time, 1 yr. 7 mo. 15 da. Amount?

9. Time, 8 yr.; rate, 3%; amount, \$ 6200. Principal?

10. Principal, \$ 7548; time, 3 mo. 5 da.; interest, \$ 119.51. Rate?

11. Principal, \$ 9000; rate, 4%; interest, \$ 632. Time?

12. Time, 2 yr. 3 mo. 20 da.; rate, 5%; amount, \$160.60. Principal?

13. Principal, $756; rate, $3\frac{1}{4}\%$; time, 3 yr. 4 mo. 20 da. Interest?

14. Principal, $120; time, 1 yr. 2 mo. 15 da.; interest, $4.35. Rate?

15. Amount, $97.57; rate, 4%; interest, $7.57. Time?

16. Time, 3 yr. 8 mo. 19 da.; rate, $4\frac{1}{2}\%$; amount, $93.39. Principal?

17. Principal, $1848; rate, $3\frac{3}{4}\%$; time, 4 yr. 9 mo. 25 da. Amount?

18. Rate, 5%; time, 4 yr. 6 mo. 23 da.; interest, $16.43. Principal?

338. Oral Exercises.

1. In what time will $100 amount to $109, at 6% interest?

2. At what rate will $200 produce $16 interest in 2 years?

3. What principal will produce $12 interest in 3 years, at 4%?

4. In what time will $300, at 4%, produce $29 interest?

5. In what time will $170 produce $1.70 interest, at 5%?

6. In what time will $360 produce $3.60 interest, at 4%?

7. In what time will $725 produce $7.25 interest, at 6%?

8. In what time will $45 produce 45¢ interest, at $4\frac{1}{2}\%$?

9. In what time will $72 produce $1.44 interest, at 6%?

10. Find the interest on $84 for 144 days, at 5%.

11. Find the interest on $125, at 5%, for 2 months 12 days.

12. At what rate will $64 produce 64¢ interest in 80 days?

13. At what rate will $40 produce $1.20 interest in 6 months?

14. A certain principal produces $120 interest at 6%. What would be the interest if the rate were 4%?

339. Written Review Exercises.

1. What number increased by 16% of itself equals 1276?

2. A capitalist sends a commission merchant $8670 to invest in cotton and to include commission at 2%. How much does the commission amount to?

3. A joiner worked on Monday 9 hr. 45 min., on Tuesday and Wednesday 10 hr. 45 min. each day, on Thursday and Friday 10 hr. 15 min. each day, and on Saturday 6 hr. 45 min. What was the average length of his day's work?

4. Thirty-two clerks are to distribute 36,000 letters on a certain day. Half of the clerks are experienced men and half of them new men. If each experienced man does twice as much as a new man, how many letters will be distributed by one of each kind?

5. Sold my house and farm of $94\frac{4}{5}$ acres for $12,300. Allowing $7000 for the house, what did I receive per acre for the land?

6. A commission merchant receives $1071 to invest in oats at 30¢ per bushel and to cover his commission at 2% for buying. How many bushels of oats does he purchase?

Should the commission merchant deduct 2% of $1071, or 2% of the cost of the oats?

7. What is the total weight of 4 hogsheads of sugar, weighing respectively $936\frac{1}{2}$, $1025\frac{9}{10}$, $846\frac{2}{5}$, and $987\frac{7}{15}$ pounds, deducting tare at 10 per cent?

8. The product of three factors is 3289; two of the factors are 23 and 11. What is the third factor?

9. A man received $2.75 per day, exclusive of Sundays, during 1903. He paid $73 for clothing for himself and family, $15 per month rent, $1.10 per day for provisions, $8 per month for fuel and light, and 25¢ per day for other expenses. How much had he left at the end of the year?

BANK DISCOUNT.

340. Thomas Tierney, wishing to borrow three hundred dollars from The Borough Bank, draws up the following promissory note:

DENVER, May 16, 1903.

Three months after date I promise to pay to the order of myself Three Hundred Dollars, value received, at The Borough Bank.

$\$ 300_{\frac{00}{100}}$. THOMAS TIERNEY.

As the note now stands, it is payable to Thomas Tierney. He transfers by *indorsing* it; that is, by writing his name on the back of the note. The effect of this indorsement is to transfer the note to the holder, in this case, the bank. As a bank requires at least a second person as a security for the payment of the loan, Mr. Tierney gets Herman A. Metz to indorse it also. By this indorsement, Mr. Metz agrees to pay the note if Mr. Tierney does not pay it at maturity, August 16.

The Borough Bank thereupon pays over to Thomas Tierney, or places to his credit on the books of the bank, the face of the note less the interest for 92 days, \$ 300 — \$ 4.60, or \$ 295.40. This interest taken in advance is called *bank discount.* The sum turned over to Thomas Tierney is called the *proceeds.*

Face of note,	\$ 300.00
Discount 92 days,	4.60
Proceeds,	\$ 295.40

To find the bank discount, compute the interest on the face of the note from the date of discount to the date of maturity.

To find the proceeds, deduct the discount from the face.

NOTE. — The usage of banks varies in different parts of the country, and the teacher should inform herself as to the local practice.

341. Written Exercises.

Find the discount at 6% on the following:

1. A 30-days note for $75.
2. 15-days note for $183.60.
3. 60-days note for $275.40.
4. 20-days note for $96.
5. 4-months note for $336.

Face of note — bank discount = proceeds.

Find the proceeds, at 7%, on

6. A 6-months note for $180.
7. A 3-months note for $36.90.
8. A 24-days note for $795.60.
9. A 90-days note for $180.
10. A 72-days note for $1000.

342. Find the discount, at 6%, on

11. A 1-month note for $600, dated Feb. 6, 1904. Due March 6, 29 days.
12. A 2-months note for $240, dated July 17, 1903.
13. A 3-months note for $360, dated April 8, 1904.
14. A 4-months note for $84, dated Dec. 24, 1905.
15. A 6-months note for $172.60, dated March 4, 1903.
16. A 60-days note for $240, dated July 17, 1904.
17. A 90-days note for $360, dated April 8, 1903.

In each of the preceding examples, it has been assumed that the note has been presented for discount the day on which it was made.

In some of the following examples, the notes are discounted at a later date, and the *term of discount* is to be ascertained; that is, the time between the date of discount and that of maturity.

The term of discount of a 30-days note dated May 1, and discounted May 19, is the time from May 19 to May 31, 12 days.

343. In the following examples, find (*a*) date of maturity; (*b*) term of discount; (*c*) discount; (*d*) proceeds.

NOTE. — The pupil that works without thinking, frequently finds the difference in time between the two dates given in the problem and uses this as the term of discount. The time between the dates given below shows in each case the time the note was *not* in the possession of the bank.

	Dated.	Face.	Time.	Discounted.	Rate.
18.	July 16, 1902;	$87.60;	30 days;	August 11, 1902;	6%

This note is due 30 days after July 16, which is August 15. If the bank discounts it August 11, 4 days before it is payable, it deducts 4 days' interest, which is 6 cents.

Answers — Date of maturity Aug. 15, 1902.
Term of discount 4 days.
Discount 6 cents.
Proceeds $87.54.

19. Date, Sept. 9, 1902; face, $124.18; time, 4 months; discounted, Nov. 18, 1902; rate, 8%.

20. Date, Dec. 5, 1902; face, $504.60; time, 30 days; discounted, Dec. 12, 1902; rate, 7%.

21. Date, Nov. 14, 1903; face, $72.36; time, 3 months; discounted, Dec. 20, 1903; rate, 6%.

22. Date, Oct. 30, 1903; face, $234; time, 90 days; discounted, Jan. 5, 1904; rate, 6%.

23. Date, Jan. 2, 1904; face, $95.90; time, 2 months; discounted, Feb. 13, 1904; rate, 6%.

24. Date, Aug. 5, 1904; face, $164; time, 60 days; discounted, Aug. 31, 1904; rate, 8%.

25. Date, Feb. 27, 1904; face, $83.20; time, 100 days; discounted, March 9, 1904; rate, 6%.

DISCOUNT OF INTEREST-BEARING NOTES.

344. Written Problems.

Brooklyn, N.Y., Oct. 16, 1904.

Sixty days after date I promise to pay to the order of John Karst, One Hundred Forty-eight $\frac{50}{100}$ Dollars, value received, with interest at 6%.

$148\frac{50}{100}$. Daniel Kelly.

1. Find the proceeds of the above note if discounted Dec. 1, 1904, at 6%.

At maturity, Dec. 15, 1904, there is due $148.50 with $1.49 interest for sixty days, a total of $149.99. If it is discounted Dec. 1, 14 days before maturity, the bank deducts 14 days' interest on $149.99, which is 35 cents, and pays over to John Karst the proceeds, $149.64, *Ans.*

To find the bank discount of an interest-bearing note, compute the interest on the amount due at maturity from the time of discount to the date of maturity.

2. Find the proceeds of a 90-days note for $175, bearing interest at 6%, discounted 33 days after date, at 6%.

3. Find the proceeds of a 60-days note for $350, bearing interest at 6%, discounted at 6%, 10 days after date.

4. Find the proceeds of a 3-months note for $840, bearing interest at 7%, discounted at bank 47 days before maturity, at 8%.

5. A 4-months note for $720, dated March 17, 1905, bearing interest at 6%, is discounted at 7%, May 10. What are the proceeds?

6. The following note was discounted at 6%, Sept. 19, 1904. Find the proceeds.

Milwaukee, Wis., June 30, 1904.

Four months after date I promise to pay Thomas Cacciola, or order, Five Hundred Dollars, value received, with interest at 6 per cent.

$500\frac{00}{100}$. George H. Greene.

345. To find the face of note, rate of discount, or term.

1. The discount at 6% on a note having 84 days to run, is $10.50. Find the face of the note.

The discount on $1 for 84 days at 6% = $1 $\times \frac{6}{100} \times \frac{84}{360}$, or $.014. If $.014 is the discount on a note for $1, $10.50 will be the discount on a note for as many dollars as $.014 is contained times in $10.50, or $750. *Ans.* $750.

2. The proceeds of a note having 84 days to run, discounted at 6%, are $739.50. Find the face of the note.

The discount on $1 for 84 days at 6% is $.014; the proceeds are $1 − $.014, or $.986. If $.986 are the proceeds of a note for $1, $739.50 will be the proceeds of a note for as many dollars as $.986 is contained times in $739.50, or $750. *Ans.* $750.

To find the **face of a note**, *divide the given discount (or proceeds) by the discount (or proceeds) of* **$1** *for the given term at the given rate.*

3. The discount on a note for $750 having 84 days to run is $10.50. What is the rate of discount?

The discount on $750 at 1% for 84 days is $750 $\times \frac{1}{100} \times \frac{84}{360}$, or $1.75. If $1.75 is produced by a rate of 1%, $10.50 will be produced by a rate of as many per cent as $1.75 is contained times in $10.50, or 6%.

To find the **rate of discount**, *divide the given discount by the discount at* **1%** *on the given sum for the given term.*

4. The discount at 6% on a note for $750 is $10.50. How many days has the note to run?

The discount on $750 at 6% for 1 day is $750 $\times \frac{6}{100} \times \frac{1}{360}$, or $.125. If $.125 is the discount for 1 day, $10.50 will be the discount for as many days as $.125 is contained times in $10.50, or 84 days.

To find the **term of discount**, *divide the given discount by the discount for* **1 day** *on the given sum at the given rate.*

346. NOTE. — To solve by the algebraic method, use x to represent the unknown quantity.

347. Written Exercises.

1. Three-months note; face, \$108; rate, 6%. Find proceeds.

2. 90-days note; face, \$360; discount, \$6.30. Find rate.

3. Proceeds, \$717.60; rate, 5%; face, \$720. Find term.

4. Discount, \$11.20; rate, 7%; term, 48 days. Find face.

5. 15-days note; face, \$1560; rate, 6%. Find discount.

6. Term, 20 days; face, \$158.40; proceeds, \$157.96. Find rate.

7. Rate, 7%; discount, \$2.10; face, \$150. Find term.

8. Two-months note; discount, \$14.70; rate, 7%. Find face.

9. For what amount must a 60-days note be drawn so that the proceeds will be \$300 when the rate of discount is 8 per cent?

10. A note for \$120 was discounted at a bank March 15, 1905. What is the date of the maturity of the note, the proceeds being \$119.52 and the rate of discount 6 per cent?

11. Find the proceeds of a 6-months note for \$875 drawn Jan. 2, 1906, and discounted at 6 per cent 35 days after that date.

12. A merchant bought 300 barrels of flour at \$4.75 per barrel, cash, and sold it for \$5 per barrel, taking in payment a 60-days note for the amount. If he has the note discounted immediately at a bank, at 7 per cent, what does he gain by the transaction?

13. What will be the face of a 30-days note, the proceeds of which when discounted at a bank at 6% will pay for 3000 bushels corn at $49\frac{3}{4}$¢ per bushel?

14. The proceeds of a note for \$1200, due March 15, 1904, and discounted at 6%, were \$1184.80. When was it discounted?

INTEREST BY ALIQUOT PARTS.

348. Written Exercises.

1. Find the interest on $387.45, for 2 yr. 8 mo. 18 da., at 7%.

	$387.45 × .07.
	$27.1215 interest for 1 yr.
	27.1215 interest for 1 yr.
6 mo. = ½ yr.	13.5607 interest for 6 mo.
2 mo. = ⅓ (of 6 mo.)	4.5202 interest for 2 mo.
15 da. = ½ (of 2 mo.)	1.1301 interest for 15 da.
3 da. = ⅕ (of 15 da.)	.2260 interest for 3 da.
Ans.	$73.68 interest for 2 yr. 8 mo. 18 da.

2. Find the interest on $432.90, at 6%, for 1 yr. 7 mo. 12 da.

	$432.90 × .06.
	interest for 1 yr.
6 mo. = ½ yr.	interest for 6 mo.
1 mo. = ⅙ (of 6 mo.)	interest for 1 mo.
10 da. = ⅓ (of 1 mo.)	interest for 10 da.
2 da. = ⅕ (of 10 da.)	interest for 2 da.
	interest for 1 yr. 7 mo. 12 da.

3. Find the amount of $874.16, at 5%, for 1 yr. 9 mo. 4 da.

	$874.16 principal.
5% = 1/20	43.708 interest for 1 yr.
6 mo. = ½ yr.	interest for 6 mo.
3 mo. = ½ (of 6 mo.)	interest for 3 mo.
3 da. = 1/30 (of 3 mo.)	interest for 3 da.
1 da. = ⅓ (of 3 da.)	interest for 1 da.
	amount for 1 yr. 9 mo. 4 da.

4. What is the amount of $95.72, for 3 yr. 6 mo. 20 da., at 5%?

	$95.72 principal.
10% = 1/10	9.572 interest for 2 yr.
1 yr. = ½ (of 2 yr.)	4.786 interest for 1 yr.
6 mo. = ½ yr.	interest for 6 mo.
20 da. = ? of 6 mo.	interest for 20 da.
	amount for 3 yr. 6 mo. 20 da.

5. Interest of $1806.45, at 4%, for 1 yr. 7 mo. 25 da.

1 yr., 6 mo., 1 mo., 15 da., 5 da., 5 da.

6. Interest for 10 mo. 29 da., at 4%, on $380.40.

$380.40 × .04.

$15.2160 interest for 1 yr.

1 mo. = $\frac{1}{12}$ yr. interest for 1 mo. } deduct from interest
1 da. = $\frac{1}{30}$ mo. interest for 1 da. } for 1 yr.

interest for 10 mo. 29 da.

7. Amount, at 6%, of $125.73, for 2 yr. 10 mo. 4 da.

8. Interest on $84.66, at 7%, for 1 yr. 4 mo. 12 da.

9. Interest, at 5%, for 4 yr. 2 mo. 7 da., on $250.

10. Amount of $1000, at 6%, for 33 days.

349. When the time is less than a year, the following facts should be remembered:

6% for a year is 1 per cent for 60 days.
5% for a year is 1 per cent for 72 days.
4½% for a year is 1 per cent for ? days.
4% for a year is 1 per cent for ? days.

11. Find the interest for 81 days, at 5%, on $876.40.

Since 5% for a year is 1% for 72 days, we have: —

72 days' interest is 1% of principal, or $8.764
9 days' interest is $\frac{1}{8}$ of 72 days, or 1.095

$9.86 interest for 81 days.

12. Amount of $954, at 4%, for 4 mo. 10 da.

Principal $954.00
3 months' interest = 1% 9.54
1 mo. = $\frac{1}{3}$ (of 3 mo.) 3.18
10 da. = $\frac{1}{3}$ (of 1 mo.)

amount for 4 mo. 10 da.

13. Interest of \$1874, at $4\frac{1}{2}\%$, for 93 da.

80 days = 1%
10 days
2 days
1 day

14. Interest of \$753.20, at 5%, for 158 days.

72 da., 72 da., 12 da., 2 da.

15. Amount of \$1234.50, for 193 days, at 6%.

60 da., 120 da., 12 da., 1 da.

16. Find the proceeds of a 90-days note, for \$873.60, at 6%.

Face	\$873.60	
60 da.	8.736	Deduct.
30 da.	4.368	
	\$860.50	proceeds.

17. Find the discount on a 3-months note, for \$1596, at 6%.

18. What are the proceeds of a 6-months note, for \$785, discounted at 6%.

19. Find the interest on \$484.40, for 1 yr. 3 mo. 17 da., at 7%.

20. Find the amount of \$683, for 3 yr. 4 mo. 11 da., at $4\frac{1}{2}\%$.

350. N.B. — Do not use unnecessary figures.

21. Principal, \$360; 5%; 3 yr. 7 mo. 18 da. Interest?

22. Principal, \$613; $4\frac{1}{2}\%$; 157 da. Amount?

23. Principal, \$1774; $3\frac{3}{4}\%$; 17 mo. 23 da. Interest?

24. Principal, \$875; 6%; 2 yr. 3 mo. 1 da. Amount?

25. Principal, \$976; 7%; 325 da. Interest?

351. By the *time* of a note is meant the number of days, etc., for which it is drawn. In these four examples the note is discounted the day it is made.

26. Face of note, $254; time, 30 days; 7%. Proceeds?

27. Face of note, $515; time, 6 months; 5%. Discount?

28. Face of note, $493; time, 60 days; 8%. Proceeds?

29. Face of note, $717; time, 15 days; 6½%. Discount?

352. Find the exact number of days. Take 360 days to year.

30. Principal, $1836.50; 6%; Jan. 2 to Dec. 1. Amount?

31. Principal, $1295.70; 7%; March 8 to April 9. Interest?

32. Principal, $765.90; 4%; Oct. 1 to Dec. 17. Interest?

33. Principal, $275.84; 5½%; May 9 to July 3. Amount?

353. By the *term* of a note is meant the number of days it has to run after it has been discounted.

34. Face of note, $100; term, 60 days; 7%. Discount?

35. Face of note, $200; term, 90 days; 6½%. Proceeds?

36. Face of note, $300; term, 24 days; 5½%. Discount?

37. Face of note, $400; term, 117 days; 8%. Proceeds?

354. In examples 38–41, inclusive, find the time by compound subtraction.

38. Principal, $25.83; 6%; Jan. 14, 1902, to Sept. 5, 1904. Interest?

39. Principal, $47.96; 5%; Feb. 6, 1903, to Aug. 1, 1906. Amount?

40. Principal, $85.30; 7%; March 25, 1904, to Jan. 13, 1907. Interest?

41. Principal, $75; 4%; April 15, 1900, to Feb. 6, 1907. Amount?

REVIEW.

355. Oral Problems.

1. Out of 500 pupils, 50 are absent. What is the per cent of attendance?

2. A can do a piece of work in 4 days; B can do it in 4 days. In what time can A and B do it, if they work together?

3. What is the interest of $1500, for 60 days, at 6%?

4. In a certain class $\frac{1}{2}$ of the pupils are under 10 years, $\frac{1}{3}$ of them are between 10 and 12, and the rest are over 12. What per cent are over 12 years?

5. If a bushel of English walnuts costs $1.60, what will 6 quarts cost?

6. A man put 5 gal. 2 qt. of syrup into bottles holding 2 quarts each. How many bottles did it require?

7. If $\frac{2}{3}$ of a yard of cloth costs $\frac{4}{5}$ of a dollar, what will $\frac{3}{4}$ of a yard cost?

8. If 9 pounds of sugar cost 48¢, what will 12 pounds cost?

9. What is the difference between $\frac{2}{7}$ of $6\frac{1}{2}$ and $\frac{3}{7}$ of $4\frac{1}{3}$?

10. How many eggs, at the rate of 15 for 25 cents, can be bought for 60¢?

11. A merchant's receipts are $1200; his gain is 20 per cent. What part of his receipts is profit?

12. If 3 men earn $72 in 8 days, how many dollars will 5 men earn in 11 days?

13. If a dealer loses 25% by selling a horse for $225, what per cent would he gain or lose by selling the horse for $325?

14. If A can do a piece of work in 2 days, B in 3 days, and C in 4 days, in what time can they do it, working together?

356. Written Problems.

1. A man sold 18 barrels sugar, each containing 306 pounds; 21 barrels, each containing 297 pounds; 5 barrels, each containing 291 pounds. What is the average weight per barrel?

2. Three men engage in a business venture. One furnishes $3000, another furnishes $5000, a third furnishes $4000. They gain $1800. What is each one's share of the profit?

What part of the money did the first furnish? What part of the profit should he receive?

3. Three ounces is what per cent of 5 pounds?

4. What is the product of $\frac{1}{8}$ of $\frac{6}{9}$ of $15\frac{1}{7}$. State the result in decimals.

5. What is $87\frac{1}{2}\%$ of $896? $896 is $87\frac{1}{2}\%$ of what sum?

6. What number is that which, diminished by $2\frac{1}{5}$, will leave $2\frac{1}{30}$.

7. How long will 200 pounds flour last 18 persons if each person is allowed $1\frac{3}{4}$ pounds per day?

8. If $\frac{3}{4}$ of $\frac{7}{8}$ of a ship cost $84,000, what is $\frac{2}{3}$ of it worth?

9. The dividend was $4689.036, the quotient .027, what was the divisor?

10. Harry Hedge earns $12 a week. He pays $4.25 for board, $0.625 for car fare, $0.375 for library fees, and $4.875 for other expenses. In how many weeks would he save $97.50.

11. For how long must $450 be at interest, at five per cent per annum, to amount to $481.62?

12. Divide 320 acres of land among A, B, and C, so that A shall have 15 acres more than B, and C shall have 27 acres more than B.

DENOMINATE NUMBERS.

357. Inductive Exercises.

1. Change 1 yd. 1 ft. to inches.
2. Change 1 yd. 1 ft. 1 in. to inches.
3. Change 49 inches to yards, feet, etc.
4. Change 49 pints to gallons, etc.
5. Add 4 lb. 8 oz. and 4 lb. 8 oz.
6. From 9 pounds take 4 lb. 8 oz.
7. Multiply 4 lb. 8 oz. by 2.
8. Divide 9 pounds by 2.
9. Divide 9 pounds by 4 lb. 8 oz.
10. How many inches in $\frac{3}{4}$ yd.?
11. How many feet and inches in .75 yd.?
12. 75 per cent of a yard = ?
13. What fraction of a yard is 27 inches?
14. Change 2 ft. 3 in. to the decimal of a yard.
15. 1 ft. 6 in. is what per cent of 2 feet?
16. Multiply 9 yd. 18 in. by 7.
17. From 18 lb. 6 oz. take 9 lb. 12 oz.

358. Troy Weight.

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweight = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

Troy weight is used in weighing gold, silver, precious stones, etc.

359. English Money.

12 pence (*d.*) = 1 shilling (*s.*)
20 shillings = 1 pound (£)

A farthing is a quarter of a penny.

REDUCTION DESCENDING.

360. Change 43 yd. 2 in. to inches.

Write 43 yd. 0 ft. 2 in., inserting the missing denomination, feet. Above 0 ft. write the number of feet in a yard, and above the 2 in. the number of inches in a foot. Since there are 3 feet in a yard, in 43 yards there are 43 times 3 feet, or 129 feet. This is written in the column of feet. Since there are 12 inches in a foot, in 129 feet there are 129 times 12 inches, or 1548 inches. Add 2 inches, making 1550 inches, which is written in the column of inches, and cancel 129 feet.

	3 ft.	12 in.
43 yd.	0 ft.	2 in.
	129 ft.	1550 in.
		Ans. 1550 in.

In working this example, 3 and 12 are used as the multipliers instead of 43 and 129. At the time the 9 of 129 is multiplied by 12, the 2 is added in, the pupil saying 12 nines are 108, and 2 are 110, writing the 0; 12 twos are 24, and 11 are 35, writing the 5; etc.

361. Written Exercises.

Change:

1. 4 yards 2 feet 8 inches to inches.
2. 2 miles 46 rods 3 yards to yards.
3. 3 pecks 5 quarts 1 pint to pints.
4. 6 bushels 3 pecks 6 quarts to quarts.
5. 2 gallons 3 quarts 1 pint to pints.
6. 7 gallons 1 quart 1 pint to pints.
7. 4 ounces 12 pennyweights 3 grains to grains.
8. 2 pounds 16 pennyweights 14 grains to grains.
9. 6 pounds 9 shillings 7 pence to pence.
10. 8 pounds 18 shillings 4 pence to pence.
11. 3 wk. 4 da. 13 hr. to hours.
12. $\frac{4}{7}$ of a week to hours.
13. $\frac{7}{8}$ of a mile to yards.
14. .25 of a rod to inches.

REDUCTION ASCENDING.

362. Change 1550 inches to yards, feet, etc.

Above 1550 in. write 12 in., the number in a foot. Dividing 1550 inches by 12 inches we obtain the quotient 129, the number of feet, and 2 inches remainder. Write the remainder in the column of inches and 129 ft. to the left of 1550 in. Reduce 129 ft. to yards, writing the result, 43 yd., as shown above, and cancelling 129 ft.

	3 ft.	12 in.
	129 ft.	1550 in.
43 yd.	0 ft.	2 in.

Ans. 43 yd. 2 in.

363. Written Exercises.

Change:

1. 4530 feet to rods, yards, etc.
2. 6324 yards to miles, rods, etc.
3. 244 pints to bushels, pecks, etc.
4. 467 quarts to bushels, pecks, etc.
5. 923 pints to gallons, quarts, etc.
6. 785 pints to gallons, quarts, etc.
7. 543 pennyweights to pounds, etc.
8. 175 grains to pennyweights, etc.
9. 625 pence to pounds, shillings, etc.
10. 836 shillings to pounds, etc.
11. 8423 min. to days, hours, etc.
12. 2348 inches to yards, etc.

ADDITION OF DENOMINATE NUMBERS.

364. Find the sum:

11 yd.	2 ft.	8 in.
	1 ft.	10 in.
4 yd.	0 ft.	5 in.
16 yd.	1 ft.	11 in.

Ans.

8 in. + 10 in. + 5 in. = 23 in. = 1 ft. 11 in. Write 11 in. and carry 1 ft.

1 ft. + 1 ft. + 2 ft. = 4 ft. = 1 yd. 1 ft. Write 1 ft. and carry 1 yd.

1 yd. + 4 yd. + 11 yd. = 16 yd. Write 16 yd.

365. Written Exercises.

Find sums:

1. 8 mi. 44 rd. 3 yd.
   6 mi. 298 rd. 4 yd.
   67 rd. 1 yd.

2. 27 rd. 3 yd. 2 ft.
   3 rd. 2 yd. 1 ft.
   78 rd. 4 yd. 2 ft.

3. 8 bu. 3 pk. 5 qt.
   16 bu. 2 pk. 3 qt.
   4 bu. 3 pk. 7 qt.

4. 243 gal. 2 qt. 1 pt.
   168 gal. 3 qt. 1 pt.
   1 qt. 1 pt.

5. 4 lb. 10 oz. 14 pwt.
   3 lb. 9 oz. 16 pwt.
   1 lb. 11 oz. 7 pwt.

6. 8 oz. 9 pwt. 21 gr.
   3 oz. 11 pwt. 6 gr.
   17 pwt. 23 gr.

SUBTRACTION OF DENOMINATE NUMBERS.

366. From 35 yd. 1 ft. 4 in.
Take 19 yd. 2 ft. 8 in.
15 yd. 1 ft. 8 in. *Ans.*

Since 8 in. is greater than 4 in., we must use 1 ft. 4 in., or 16 in., as the minuend. 16 in. − 8 in. = 8 in. As the minuend now contains 0 ft., 1 yd. is taken from 35 yd. Changing the yard to 3 ft., and deducting 2 ft., leaves 1 ft 34 yd. − 19 yd. = 15 yd.

367. Written Exercises.

Find differences:

1. 183 rd. 4 yd. 1 ft.
   68 rd. 5 yd. 2 ft.

2. 91 mi. 83 rd. 2 yd.
   26 mi. 122 rd. 4 yd.

3. 3 pk. 1 qt.
   1 pk. 4 qt. 1 pt.

4. 29 gal. 2 qt.
   28 gal. 3 qt. 1 pt.

5. 8 lb. 3 oz. 8 pwt.
   6 lb. 8 oz. 10 pwt.

6. £24 6*s.* 3*d.*
   £3 9*s.* 8*d.*

MULTIPLICATION OF DENOMINATE NUMBERS.

368. Multiply $\frac{34 \text{ yd. } 2 \text{ ft. } 9 \text{ in.}}{244 \text{ yd. } 1 \text{ ft. } 3 \text{ in.}}$ by 7.

7 times 9 in. = 63 in. = 5 ft. 3 in. Write 3 in. 7 times 2 ft. = 14 ft. Carry 5 ft., making 19 ft., or 6 yd. 1 ft. Write 1 ft. Multiply 34 yd. by 7, adding in 6 yd. when the 4 is multiplied.

369. Written Exercises.

Find products:

1. 6 mi. 24 rd. 4 yd. by 9.
2. 36 rd. 4 yd. 2 ft. by 12.
3. 24 bu. 3 pk. 6 qt. by 14.
4. 2 pk. 3 qt. 1 pt. by 36.
5. 11 gal. 2 qt. 1 pt. by 8.
6. 3 gal. 1 qt. 1 pt. by 32.
7. 8 lb. 4 oz. 12 pwt. by 10.
8. 16 oz. 12 pwt. 20 gr. by 4.
9. £4 12*s.* 6*d.* by 20.
10. £28 16*s.* 9*d.* by 7.

DIVISION OF DENOMINATE NUMBERS.

370. Divide $\frac{244 \text{ yd. } 1 \text{ ft. } 3 \text{ in.}}{34 \text{ yd. } 2 \text{ ft. } 9 \text{ in.}}$ by 7. *Ans.*

The quotient of 244 yd. divided by 7 is 34 yd., with a remainder of 6 yd. Reducing 6 yd. to ft. and adding in 1 ft., the dividend is 19 ft. 19 ft. ÷ 7 = 2 ft. with 5 ft. remainder. 5 ft. 3 in. = 63 in. 63 in. ÷ 7 = 9 in.

371. Written Exercises.

Find quotients:

1. 44 mi. 124 rd. 2 yd. by 8.
2. 14 yd. 1 ft. 9 in. by 21.
3. 37 bu. 1 pk. 2 qt. by 6.
4. 12 bu. 3 pk. 6 qt. by 18.
5. 7 gal. 3 qt. 1 pt. by 3.

6. 3 gal. 1 pt. by 5.
7. 28 lb. 10 oz. 16 pwt. by 24.
8. 4 oz. 10 pwt. 3 gr. by 9.
9. £ 24 17*s.* 4*d.* by 16.
10. £ 3 7*s.* 6*d.* by 10.

372. Divide 244 yd. 1 ft. 3 in. by 34 yd. 2 ft. 9 in.

In dividing one concrete number by another concrete number, the divisor and the dividend must be of the same denomination. Thus, to divide \$2 by 25¢, we change the dividend to cents, 200 cents ÷ 25 cents, or the divisor to dollars, \$2 ÷ \$¼. The quotient is 8, an abstract number; that is, 25 cents is contained in 200 cents 8 times.

244 yd. 1 ft. 3 in. = 8799 in.; 34 yd. 2 ft. 9 in. = 1257 in. 8799 in. ÷ 1257 in. = 7, *Ans.*

The result would be the same if we divided $733\frac{1}{4}$ ft. by $104\frac{3}{4}$ ft., or $244\frac{5}{12}$ yd. by $34\frac{11}{12}$ yd.

373. Written Exercises.

Find quotients:

1. 4 mi. 36 rd. 1 yd. by 6 rd. 3 yd.
2. 88 rd. 2 yd. 2 ft. by 8 rd. 4 yd. 2 ft.
3. 21 bu. 2 pk. 4 qt. by 1 bu. 3 pk. 4 qt.
4. 15 bu. 1 pk. by 3 pk. 6 qt. 1 pt.
5. 60 gal. 1 pt. by 4 gal. 2 qt. 1 pt.
6. 16 gal. 3 qt. 1 pt. by 2 qt. 1 pt.
7. 17 lb. 11 oz. 10 pwt. by 8 lb. 11 oz. 15 pwt.
8. 1 lb. 2 oz. 18 pwt. by 3 oz. 14 pwt. 12 gr.
9. £24 16*s.* 8*d.* by £18 12*s.* 6*d.*
10. £2 14*s.* 3*d.* by £8 2*s.* 9*d.*

374. Oral Problems.

1. What will be the cost of 3 lb. 7 oz. of tea, at 64¢ per pound?

2. How many feet in $2\frac{1}{4}$ rods?

3. At $37\frac{1}{2}$¢ per peck, what shall I receive for 4 bushels of potatoes?

4. What will be the cost of a ton of hay at $97\frac{1}{2}$¢ per cwt.?

5. If slate pencils cost 2 mills each, how many can be bought for $4?

6. At $5.00 per ton, how many pounds of coal can be bought for 1¢?

7. Find the cost of 3 T. 480 lb. coal at $5 per ton.

8. At $5 per ton, how many tons and pounds of coal can I buy for $10.80?

9. Find the cost of 4 yd. 1 ft. of ribbon, when 2 yd. 2 ft. cost 40 cents.

10. In $2\frac{3}{4}$ pecks, how many quarts?

11. How many hours in $\frac{5}{8}$ of a day?

12. 1.25 pecks are how many quarts?

13. At $12 per ounce, what is $\frac{5}{6}$ of a pound of gold worth?

14. How many feet in a quarter of a mile?

15. How many tablespoons, each weighing 2 ounces, can be made from 2 lb. 10 oz. of silver?

375. Written Problems.

1. What will be the cost of 150 yards silk at 3/6 per yard? $3/6 = 3s.\ 6d.$, read three and sixpence.

2. If £ 1 = $ 4.8665, what will be the cost in U. S. money of 75 books at 18 pence each?

3. A merchant sells 37 coats at £ 3 5*s.* each, less 10%. What is the amount of his bill in English money?

4. Find 25% of £ 183 14*s.* 8*d.*

5. A silver dollar weighs 412½ grains. How many ounces of pure silver are there in 1000 silver dollars if the coin is $\frac{9}{10}$ pure silver?

6. The wheels of an engine being 16 ft. 8 in. in circumference, and the number of revolutions 150 per minute, how far does it go in an hour? Give answer in miles and rods.

7. What fractional part of 30 rd. 5 yd. 1 ft. is 8 rd. 4 yd. 2 ft.?

8. What decimal part of a mile is 39.27 yd.?

9. 3 bu. 1 pk. 5 qt. is what per cent of 20 bu. 1 pk. 6 qt.?

10. If a letter-carrier in delivering letters takes 47,520 steps in a day, each step averaging 20 inches, how many miles does he walk?

11. 43 gal. 3 qt. 1 pt. alcohol are sold for $ 70.20. What is the price per gallon?

12. After taking out 15% of the grain in a bin, there remained 40 bu. 3 pk. 5 qt. How many bushels were there at first?

13. A merchant bought 51 tons 17 cwt. 3 qr. 25 lb. of wool, and sold 27 tons 4 cwt. 2 qr. 27 lb. Of the remainder, one-half was lost by fire. How much had he left?

28 lb. = 1 quarter; 4 quarters = 1 cwt.

14. An invoice of wool weighs 32 tons 17 cwt. 2 qr. 11 lb. State the value in £ *s.* *d.*, at 10*d.* sterling per pound.

1 ton = 2240 lb.

15. How many minutes in February, 1904?

16. If a locomotive runs 25 mi. 48 rd. in 50 minutes, how far will it run in 12 hours?

Give answer in miles and decimals of a mile.

17. I wish to put 111 bu. 2 pk. 4 qt. of grain into 47 bags. What quantity must each contain?

18. If a river current carries a raft of lumber at the rate of 4 miles 180 rods per hour, how long will it take the raft to float 365 miles?

19. Bought 28,500 pounds of hay at $\$12\frac{1}{2}$ a ton, and sold it at $\$0.87\frac{1}{2}$ per hundredweight. What was the gain?

376.

1 pound Troy = 5760 grains.
1 pound Apothecaries' = 5760 grains.
1 pound Avoirdupois = 7000 grains.

How many grains in a Troy ounce? In an Avoirdupois ounce?

1. Find the value of a dozen silver spoons, each weighing 3 oz. 5 pwt., at $\$1.20$ per oz.

2. A gold chain weighs 384 grains. What is its cost at $\$1.15$ per pwt.?

3. Add 4 lb. 6 oz. 18 gr., 5 oz. 9 pwt., 3 lb. 20 gr., and 9 lb. 11 oz. 15 pwt. 5 gr.

4. How many spoons, each weighing 2 oz. 18 pwt., can be made from 5 lb. 9 oz. 12 pwt. silver?

5. What fraction of a pound Avoirdupois is a pound Troy? What per cent of an ounce Avoirdupois is a Troy ounce?

6. What is the value, at $\$1.60$ per oz. Troy, of a silver pitcher weighing 4 lb. 8 oz. Avoirdupois?

7. At $60¢$ per ounce, what is the value of the silver contained in a half-dollar, which weighs 192.9 grains, $\frac{9}{10}$ being pure silver?

8. What per cent of a lb. Avoirdupois is a Troy pound?

MISCELLANEOUS.

377. Oral Problems.

1. If 4 books cost \$ 1.25, what will a dozen cost?

2. If 3 pounds of sugar cost $16\frac{1}{2}\not{c}$, what will be the cost of 50 pounds?

1 pound costs $5\frac{1}{2}\not{c}$, etc.

3. If 48 pounds of tea cost \$ 20, what will 12 pounds cost?

12 pounds will cost $\frac{1}{4}$ of \$ 20.

4. Bought 17 yards of cloth for \$ 30. How many yards could I have bought for \$ 90?

5. If 36 men do a piece of work in 105 days, how long will it take 72 men to do it?

6. If 7 railway trucks weigh 14 tons, how much would 29 trucks weigh?

7. How long will it take 8 horses to plough a field, if 3 horses can do it in 8 days?

8. What is the height of a steeple that casts a shadow of 300 feet, if an 8 foot pole casts a shadow of 12 feet.

9. If 18 men mow 90 acres of grass in 5 days, how many acres will 36 men mow in 5 days? In 10 days?

10. If 60 yd. carpet $\frac{3}{4}$ yard wide will cover a floor, how many yards $\frac{5}{8}$ yard wide will be required?

378. Written Problems.

1. A piece of cloth, measured with a yard measure that is 1 inch too short, appears to be 25 yards long. What is its true length?

2. Exchanged 40 yd. muslin, worth $10\frac{1}{2}\not{c}$ per yard, for 15 yards linen. What is the value of the linen per yard?

3. If 3 men or 6 women can do a piece of work in 56 days, in what time will 1 man and 2 women working together do it?

4. If 5 men can do as much in a day as 8 boys, how long will it take 32 boys to finish a piece of work which 15 men can do in 12 days?

5. If \$100 gain \$4 in 1 year, what will \$350 gain in $3\frac{1}{2}$ years?

6. If 48 horses in 10 days consume 180 bushels oats, how many bushels will 32 horses consume in 10 days? In 12 days? In 15 days?

7. If 5 men mow 45 acres of grass in 6 days, in how many days will 12 men mow 90 acres?

379. If 5 men mow 45 acres in 6 days,

1 man will mow 45 acres in 6 days $\times$ 5.

1 man will mow 1 acre in $\frac{6 \text{ days} \times 5}{45}$.

12 men will mow 1 acre in $\frac{6 \text{ days} \times 5}{45 \times 12}$.

12 men will mow 90 acres in $\frac{6 \text{ days} \times 5 \times 90}{45 \times 12}$.

Cancelling, $\frac{\cancel{6} \text{ days} \times 5 \times \overset{2}{\cancel{90}}}{\underset{\cancel{2}}{\cancel{45}} \times \cancel{12}} = 5$ days, *Ans.*

380. In practice, the work is somewhat shortened. Since the number of days is required, we write the given number of days last, with a line underneath.

$$\left.\begin{array}{l} 5 \text{ men mow } 45 \text{ acres} \\ 1 \text{ man mows } 1 \text{ acre} \\ 12 \text{ men mow } 90 \text{ acres} \end{array}\right\} \begin{array}{l} \text{days.} \\ \dfrac{6 \times 5 \times 90}{45 \times 12}. \end{array}$$

If 5 men do the work in a certain time, 1 man will require 5 times as many days. We place 5 in the numerator (as a multiplier). To

cut 1 acre, he will take $\frac{1}{45}$ of the time required to cut 45 acres. Place 45 in the denominator (as a divisor).

12 men will take $\frac{1}{12}$ of the time 1 man requires. Place 12 in the denominator. To cut 90 acres will require 90 times as long. Place 90 in the numerator.

8. If 12 horses eat 60 bushels of oats in 6 days, how many bushels will 24 horses eat in 3 days?

Make bushels the last term.

12 horses in 6 days eat
1 horse in 1 day eats
24 horses in 3 days eat } bu.
60

9. If 24 men use 240 pounds of beef in 2 weeks, how many pounds will 18 men use in 8 weeks?

24 men in 2 weeks use 240 lb.

10. If 6 printers can print 1656 sheets in 9 days, how many sheets will 15 printers print in 10 days?

11. How much will it cost to feed 520 sheep for 36 days, if it costs $128 to feed 160 sheep 48 days?

12. In what time will 8 masons build a wall 84 feet long, working 10 hours a day, if 12 masons build a wall 96 feet long in 8 days, working 8 hours a day?

13. How much money must I lend for 1 year and 3 months, when the rate of interest is 5 per cent, in return for $60 lent me for 9 months, which I borrowed at 4 per cent?

14. If 27 men build 54 rods of wall in 6 days, how many rods will 32 men build in 9 days?

15. If 50 men can do a piece of work in 90 days, working 8 hours a day, in how many days will 72 men do it, working 10 hours a day?

16. If $350 earns $42 interest in 3 years, how much will $225 earn in 5 years?

17. If a wall 34 feet high could be built by 68 men in 15 days, how many men could build a wall 32 feet high in 8 days?

18. If a ship's crew of 500 men have provisions to serve for 48 days, at the rate of 27 ounces a day for each man, how many men will the same provisions serve for 60 days, allowing each man 30 ounces a day?

19. How many hours a day must 9 men work so that they may do as much in 16 days as 12 men can do in 15 days of 8 hours each?

20. If 30¢ is paid for 6 lb. 14 oz. of bread, when wheat is $85\frac{1}{2}$¢ per bushel, what should be paid for 23 lb. 12 oz., when wheat is 99¢ per bushel?

21. If 3 men can do as much work as 7 boys, how long will it take 28 boys to do as much work as 16 men can do in 24 days?

22. A crew of 16 men have provisions for 36 days, allowing 20 ounces to each man per day. After sailing 10 days they pick up 10 shipwrecked sailors. How long will the provisions then last at the rate of 16 ounces per man?

23. If A can do a piece of work in 4 days, and B can do the same work in 9 days, how many days will it take both, working together?

A can do $\frac{1}{4}$ of the work in one day, and B $\frac{1}{9}$ of it. Together they can do $\frac{1}{4} + \frac{1}{9}$, or $\frac{9}{20}$ in one day. If they do 9 twentieths in one day, to do 20 twentieths, or the whole work, will require $(20 \div 9)$ days, or $2\frac{2}{9}$ days.

24. If one man can do a piece of work in 24 days, and another man can do it in 48 days, how long will it take both, working together?

APPROXIMATIONS.

Pupils should be drilled to take a broader view of their work, by estimating the probable result before taking a pencil. In this way many absurd answers might be avoided.

381. Give approximate answers at sight:

1. Find the interest of \$ 150, at 4%, from Jan. 1, 1903, to Dec. 30, 1905. (Nearly 3 years.)

2. What is the weight, at $57\frac{1}{2}$ lb. per cubic feet, of a cake of ice 4 ft. by 2 ft. by $1\frac{1}{2}$ ft? (Nearly 60 lb. per cubic feet.)

3. Find the amount of goods sold, the commission at $2\frac{7}{8}\%$ being \$ 11.75. (About 3%.)

4. What % of 497 is 249?

5. What % of $3\frac{15}{16}$ is $11\frac{81}{82}$?

6. Cost of 19,987 ft. boards at \$ 30.05 per M?

7. How much will be paid for 4 barrels sugar, each containing 299 pounds, at $5\frac{1}{16}¢$ per pound?

8. $18.0327 \div 4.5026$.

9. $83\frac{19}{20} \div 3\frac{29}{30}$.

10. 74 A. 155 sq. rd. land at \$ 79 per acre?

11. $487\frac{3}{4}$ is what per cent of 960?

12. If 17 bu. 37 lb. of corn cost \$ 8.75, what will 52 bushels cost?

13. About how many cords of wood in a pile 25 feet long, 4 feet wide, 5 feet high?

14. How many bushels ($1\frac{1}{4}$ cu. ft.) can be placed in a bin 6 feet long, 5 feet wide, 4 feet high?

15. How many acres in a field 52 rods long, 30 rods wide?

16. About how many yards are there in the side of a square field containing 1 acre (4840 square yards)?

REVIEW OF SIMPLE NUMBERS.

382. Written Exercises.

6748
× 427
47236
283416
2881396

After multiplying by 7, the pupil multiplies this latter product by 6 tens, which gives him the product by 42 tens. In this way one line is saved.

383. Find products:

1. $3{,}925 \times 328$
2. $12{,}345 \times 273$
3. $2{,}087 \times 287$
4. $20{,}308 \times 142$
5. $4{,}321 \times 189$
6. $31{,}265 \times 164$
7. $5{,}763 \times 426$
8. $87{,}093 \times 486$
9. $6{,}905 \times 364$
10. $64{,}271 \times 357$

3289
832
26312
105248
2736448

First multiply by 800, by placing the first figure of the product by 8 in the hundreds' place. Multiply this by 4, writing the first figure in the units' place.

11. 4008×214
12. 8736×742
13. 3764×327
14. 1087×848
15. 8319×416
16. 6352×927
17. 2781×525
18. 9060×1166
19. 6329×618
20. 2345×1272

21. Multiply 6984 by 25. $\frac{1}{4}$ of 698400.
22. 4327×75
23. $3762 \times 62\frac{1}{2}$. Multiply 376,200 by $\frac{1}{8}$
24. $5796 \times 62\frac{1}{2}$
25. $8383 \times 12\frac{1}{2}$
26. $3428 \times 37\frac{1}{2}$
27. $7154 \times 87\frac{1}{2}$
28. $6419 \times 33\frac{1}{3}$
29. $6208 \times 66\frac{2}{3}$

REVIEW OF FRACTIONS.

384. Written Exercises.

$7854 \times \frac{3}{4}$	$9365 \times \frac{7}{8}$
$1963\frac{1}{2}$ Deduct $\frac{1}{4}$.	$1170\frac{5}{8}$ Deduct $\frac{1}{8}$.
$5890\frac{1}{2}$ *Ans.*	$8194\frac{3}{8}$ *Ans.*

Multiply 6578 by $9\frac{2}{3}$.

$$\begin{aligned} 65{,}780 &= 10 \text{ times number.} \\ 2{,}192\tfrac{2}{3} &= \tfrac{1}{3} \text{ number (deduct).} \\ 63{,}587\tfrac{1}{3} &\ \textit{Ans.} \end{aligned}$$

385. Find products:

1.	$176 \times \frac{15}{16}$	11.	$4844 \times 9\frac{1}{2}$
2.	$273 \times \frac{12}{13}$	12.	$8960 \times 8\frac{7}{8}$
3.	$4554 \times \frac{8}{9}$	13.	$3245 \times 7\frac{7}{8}$
4.	$1001 \times \frac{10}{11}$	14.	$9060 \times 11\frac{4}{5}$
5.	$3243 \times \frac{4}{5}$	15.	$658 \times 99\frac{1}{2}$
6.	$6776 \times \frac{5}{6}$	16.	$658 \times 99\frac{3}{4}$
7.	$2307 \times \frac{6}{7}$	17.	$725 \times 119\frac{4}{5}$
8.	$7284 \times \frac{7}{8}$	18.	$347 \times 79\frac{5}{6}$
9.	$5631 \times \frac{9}{10}$	19.	$418 \times 89\frac{1}{2}$
10.	$9657 \times \frac{11}{12}$	20.	$543 \times 49\frac{2}{3}$

386. Written Exercises.

Note. — Do not use too many figures.

1. Add $\frac{4}{5}$, $2\frac{1}{4}$, $\frac{3}{7}$, $\frac{5}{8}$.
2. Divide each of the following fractions by 6:

$$\tfrac{5}{7}, \tfrac{12}{18}, \tfrac{25}{46}, \tfrac{84}{103}.$$

3. Reduce $\frac{7}{8}$ of $\frac{6}{11}$ of $\frac{5}{12}$ of $2\frac{2}{7}$ to a simple fraction
4. $38\frac{5}{9} - 21\frac{10}{12}$. $40\frac{2}{3} - 18\frac{7}{8}$

5. What fraction of £1 18*s.* 9*d.* is 5*s.* 6*d.*?

6. Multiply $24\frac{1}{3}$ by $\frac{4}{5}$ of $\frac{3}{7}$.

7. What is the greatest common divisor of 657 and 1168? the least common multiple of 12, 16, 20, 30?

8. What must be taken from $8\frac{5}{16}$ to leave $3\frac{7}{12}$?

9. Reduce $\frac{432}{528}$ and $\frac{4083}{6663}$ to their lowest terms.

10. Which is the greatest and which is the least, $\frac{1}{2}$ of $\frac{4}{5}$, $\frac{2}{8}$ of $\frac{5}{8}$, and $2\frac{1}{3}$ of $\frac{2}{11}$?

11. What must be added to $3\frac{4}{11}$ to make $5\frac{4}{9}$?

12. Add $\frac{3}{8}$ of a week, $\frac{5}{6}$ of an hour, $\frac{7}{12}$ of a minute.

13. How much is 9 times each of the following fractions?

$$\frac{5}{8},\ \frac{6}{11},\ \frac{7}{27},\ \frac{19}{68}.$$

14. $30\frac{2}{7} \div \frac{3}{8}$ of 7.

15. $\frac{7}{12} + \frac{5}{6}$ of $\frac{1}{10} + \frac{5}{8}$ of $\frac{2}{3}$.

16. What part of a 10-acre field is 4 A. 100 sq. rd.?

17. What is the least number that will contain each of the numbers 6, 15, 18, and 20?

18. What must be multiplied by $4\frac{1}{3}$ to produce $16\frac{1}{4}$?

19. What is the value of $\dfrac{\frac{1}{2} + \frac{3}{7}}{4\frac{1}{4}}$?

20. What quantity must be divided by $4\frac{2}{7}$ to produce $8\frac{5}{9}$?

21. Find the value of $\dfrac{2\frac{1}{4} \div \frac{3}{5}}{4\frac{1}{12} - \frac{5}{6}}$.

22. How much is $\dfrac{\frac{3}{4} - \frac{2}{3}}{2\frac{1}{6}}$ of 3 da. 15 hr. 32 min.?

23. Reduce $\frac{4}{15}$ mile to rods.

24. Add $\frac{2}{3}$, $\frac{4}{9}$, $5\frac{1}{6}$. Subtract $4\frac{5}{12}$ from the sum.

25. Multiply $\frac{3}{4}$ of $5\frac{1}{8}$ by $7\frac{1}{5}$. Divide the result by $1\frac{4}{5}$.

REVIEW OF DECIMALS.

387. Written Exercises.

1. Express as decimals $\frac{57}{100}$, $\frac{7}{1000}$, and $\frac{27}{320}$.

2. $.395 + 86.7 + 209.0043 + .81 + 3.075 + 27$.

3. Divide 34,020.072 by 5.309. $570 \div .005 = ?$

4. Multiply 80.037 by 10. Seventy-three hundred-thousandths by one hundred. $.2054 \times 1000 = ?$

5. Subtract 48.8067 from 53.07. $.0539 \times 26.08 = ?$

6. The smaller of two numbers is 8.5307, and their sum is 25.07. Find the larger number.

7. Express .39, 6.175, .00036, and 74.0005 as common fractions (or mixed numbers).

8. Divide .826 by 100; 543.71 by 10,000; and fifty-nine ten-thousandths by one thousand.

9. Find the difference between 9.84 and 38.005, and the continued product of 83.09, .734, and 5.007.

10. Reduce 6 shillings 9 pence to the decimal of a pound sterling.

11. Express as decimals seven hundredths, forty-three ten-thousandths, and ninety-one millionths.

12. Change $\frac{1}{16}$, $8\frac{1}{80}$, $\frac{1}{125}$, and $\frac{1}{256}$ into decimals. Find their sum.

13. Express .42796 as a common fraction, and the sum of $\frac{7}{10}$, $\frac{8}{100}$, and $\frac{873}{10000}$ as a decimal.

14. $3.009 \times .07 \times .0907$.

15. Divide .0075 by .15, and .00044408 by .0112.

16. Divisor, 403.6; quotient, 2.709. Dividend?

17. What is the value of $\frac{.035 \times .0056}{.00007}$?

18. Change 69 rods to the decimal of a mile.

19. Change .4285 month (30 days) to days, hours, etc.

SPECIAL DRILLS.

388. Give sums:

1. $856 + 256 = 856 + 200 + 50 + 6$

The pupil says (or thinks) only 1056, 1106, 1112.

2. $576 + 425$
3. $685 + 599$
4. $749 + 312$
5. $567 + 658$
6. $\$6.73 + \$3.94$
7. $\$8.27 + \$4.89$

Give remainders:

8. $1244 - 655 = 1244 - 600 - 50 - 5$

Think only 644, 594, 589.

9. $1021 - 576$
10. $1264 - 685$
11. $1040 - 312$
12. $1322 - 643$
13. $\$12.00 - \$8.73$
14. $\$11.05 - \$2.69$

Give products:

15. $24 \times 21 = 20$ times $24 + 24 = 480 + 24$

Say only 480, 504.

16. 33×21
17. 22×31
18. 41×41
19. 32×41
20. 31×31
21. 44×21

Give sums:

22. $425 + 99 = 425 + 100 - 1$

Say only 525, 524.

23. $576 + 99$
24. $999 + 425$
25. $\$8.68 + \$4.99$

Give remainders:

26. $565 - 99 = 565 - 100 + 1$

Say only 465, 466.

27. $743 - 99$
28. $1230 - 999$
29. $\$12.13 - \$4.99$

Give products:

30. $27 \times 99 = 100$ times $27 - 27 = 2700 - 27$
31. 36×99
32. 24×99
33. 98×99

389. Oral Review Problems.

1. What will be the cost of 48 yards of cloth at $87\frac{1}{2}¢$ per yard?

2. A horse was sold for $80, which was $\frac{1}{3}$ of the cost. How much was lost on the horse?

3. How many yards of carpet 27 inches wide will be needed to cover a floor containing 48 square yards?

4. Paid $3.45 for groceries, $1.50 for dry goods, and 99¢ for sundries. What is the total?

5. From a chest containing $25\frac{1}{4}$ pounds of tea, $8\frac{1}{2}$ pounds were sold. How many pounds remain?

6. What would be the cost of 2 bushels blueberries at 5¢ per quart?

7. $83\frac{1}{4}$ yards of cloth are divided into 9 pieces. How many yards are there in each piece?

8. I buy hardware to the amount of $6.37. I give the storekeeper two $5 bills. How much change should I receive?

9. What will be the cost of 24 yards of calico at $4\frac{3}{4}¢$ per yard?

10. What should I pay for 19 baseballs at $1.25 each?

11. At $1.87\frac{1}{2}$ per yard, what will be the cost of 120 yards of silk?

12. For $120, how many yards of silk can I buy at $1.87\frac{1}{2}$ per yard?

13. What is the interest of $300, for 30 days, at 6 per cent?

14. What will 18 oranges cost at 35¢ per dozen?

15. At $4\frac{3}{4}¢$ per yard, how many yards of calico can I buy for 95¢?

16. How many square yards are there in a field 41 yards long, 42 yards wide?

17. If I pay 15¢ for $3\frac{1}{3}$ yards of muslin, what is the price per yard?

18. How many acres of land are there in two farms containing, respectively, 347 and 495 acres?

19. At $87\frac{1}{2}$¢ each, how many baseballs can be bought for $56?

20. How much will be paid for 21 pounds butter, at 28¢ per pound?

21. Paid 23¢ for calico, 27¢ for ribbon, and 48¢ for collars. What was the amount of my bill?

22. A farmer had 95 sheep. He sold 39, and 17 died. How many had he left?

23. What will be the cost of 16 baseballs, at 49¢ each?

24. How much paint will there be in 27 casks, each containing 75 pounds?

25. A man divided a 429-acre farm into plots of 13 acres each. How many such plots were there?

26. There are 900 men in a certain regiment. How many companies of 75 men each are in the regiment?

27. Find the cost of 136 pounds sal-soda, at $\frac{7}{8}$¢ per lb.

28. At $19\frac{1}{2}$¢ per yard, what will be paid for 64 yards gingham?

29. How many square inches in a sheet of paper $10\frac{1}{2}$ inches long by $4\frac{1}{2}$ inches wide?

30. If $2\frac{2}{3}$ yards of cloth are needed for a jacket, how many jackets can be made from $18\frac{2}{3}$ yards?

31. How many yards around a field 96 yards long, 75 yards wide?

32. What will be the area, in square rods, of a triangle 33 rods base, altitude 42 rods?

33. How many acres in 4960 square rods?

34. How many feet in a mile?

35. I paid $16.25 for cloth at $1.25 per yard. How many yards did I buy?

36. Half a number + $\frac{1}{3}$ of the same number = 85. What is the number?

37. I mix 4 pounds of coffee costing 20¢, with 6 pounds costing 25¢. What is the mixture worth per pound?

38. A tailor makes up 99 yards of cloth into trousers, using $2\frac{3}{4}$ yards per pair. How many pairs of trousers does he make?

39. At 60¢ per pound, what will be the cost of a chest of tea weighing 45 pounds?

40. A man owns a strip of land with a frontage of 576 feet. How many lots 18 feet front can he make?

41. A can do a piece of work in 5 hours, B in 7 hours. How long will it take both working together?

42. At what rate will $300 gain $24 in 2 years?

43. What sum of money will gain $30, in 2 yr. 6 mo., at 6%?

44. If a staff 12 feet long casts a shadow of 3 feet, what is the length of a pole that casts a shadow of 27 feet at the same time?

45. If 20 men can perform a piece of work in 8 days, how many men will it take to do the same work in 5 days?

46. An agent receives $8200 to invest after deducting his commission of $\frac{1}{40}$ of the amount invested. What is the agent's commission?

47. A lot is sold for $1200, at a loss of 20 per cent. What part of $1200 is the loss?

390. Written Problems.

1. A rug costs \$ 20. It is sold at a profit of 20%. The selling price is 20% below the marked price. How much is received for the rug? What is the marked price?

2. What price must cloth, which cost \$ 2 per yard, be marked so that a profit of 20% will be made when the cloth is sold at 20% less than the marked price?

3. A coal bin is 6 feet long and 4 feet wide. How deep must it be to contain 5 tons of stove coal, if one ton occupies 36 cubic feet of space?

4. A man walking at the rate of 3 mi. 96 rd. per hour will walk how far in 3 hr. 16 min.?

5. If a merchant pays $6\frac{1}{4}¢$ per yard for muslin, and sells the same for $7\frac{1}{4}¢$ per yard, what is his gain per cent?

6. Make and solve a problem illustrating the application of percentage to the finding of an agent's commission.

7. Multiply eight hundred (units) and forty-six ten-thousandths by three thousand forty millionths.

8. What is the interest on \$ 128.40, for 1 yr. 5 mo. 17 da. at 6 per cent?

9. A regiment of 940 men, during the war, lost 532 of their number by death and 125 by desertion. What was the percentage of loss in each case, and what per cent remained for service?

10. A merchant sold a lot of damaged sugar at a loss of 25 per cent, receiving \$ 1972.65. How much did the sugar cost him?

11. What is a pile of wood 15 feet long, $10\frac{1}{2}$ feet high, and 12 feet wide worth, at $\$ 4\frac{1}{2}$ per cord?

(1 cord = 128 cu. ft.)

12. Add the greatest and the least of the three fractions $\frac{17}{19}$, $\frac{6}{7}$, $\frac{19}{28}$; and divide the sum by the remaining fraction.

13. Multiply 82 ten-thousandths by 7 and 5 hundredths, and divide the product by 705 millionths.

14. Find the cost of 96 feet of pine lumber at $25 per M, and 1650 laths at $3 per M.

15. A horse costing $160 is sold for $180. What is the gain per cent? What is the loss per cent when a horse costing $180 is sold for $160?

16. A merchant sold 600 barrels of flour for $3450, at a loss of $4\frac{1}{6}$ per cent. What did the flour cost him per barrel?

17. How long would it take a person to count a million silver dollars, at the rate of 100 a minute, and working 8 hours a day?

18. Find the number of days from March 2, 1903, to August 11, 1903.

19. Find the interest on a note for $250, dated Jan. 21, 1904, and paid May 30, 1904, at 6 %.

20. Divide 22.5 by 51.75, and express the result in the form of a fraction.

21. By the census of 1890, the population of a certain city was 26,275. By the census of 1900, its population was 31,530. Find the per cent of increase.

22. Each of two boys bought 100 apples for a dollar. The first boy sold his, 4 apples for 5¢; the second sold his, 5 apples for 6¢. Which boy gains the more per cent? How much more?

23. A quantity of coal was bought for $900. For what must it be sold to gain $33\frac{1}{3}$ %?

24. By selling a house for $5760, a man gained on the cost 25 %. What was the cost?

25. Change to other methods of expression, $\frac{1}{4}$, $\frac{1}{8}$, $.37\frac{1}{2}$, $\frac{5}{4}$, $.16\frac{2}{3}$.

26. A note of $1260, dated July 5, 1904, was paid June 7, 1906, with interest at 8%. What was the amount paid?

27. A flock of sheep has been increased by 250% of its number, and now numbers 1050. What was the original number?

28. Bought a house for $6240, and sold it so as to gain 35%. What did I sell it for?

29. Sold goods at a loss of 20%, an actual loss of $57.50. What was the first cost?

30. The milk from a herd of 25 Jersey cows, sold at 6¢ a quart, amounted in one summer to $2025. How many quarts were sold, and what was the average quantity from each cow?

31. A woman has three children. She pays for each $15 a year for having his clothes made, $1.50 a month for his mending, and $0.35 a week for his washing. How much could she save in a year if she knew how to wash, make clothes, and mend?

32. A farmer exchanged 340 bushels of corn worth 75¢ per bushel, for barley worth $1 per bushel, and oats worth 50¢ per bushel. How many bushels of each did he receive, the quantity of barley and oats being equal?

33. A pole stands $\frac{1}{7}$ in the mud, $\frac{2}{5}$ in the water, and 32 ft. in the air. How long is the pole?

34. Bought flour for $8.25, and sold it for $9. What is the per cent of gain?

35. Bought flour for $9 and sold it for $8.25. What is the per cent of loss?

36. If two-thirds of a yard of silk can be bought for $\$\frac{3}{4}$, how many yards can be bought for $\$3\frac{3}{8}$?

37. A drover sold 250 sheep for $1150, which was 15% more than they cost. What was the cost of each sheep?

38. Find a common divisor of 72 and 90.

39. How many feet of paper, 18 inches wide, will paper the sides of a room 16 feet by 14 feet, and 10 feet high, deducting 174 square feet for doors and windows?

40. Find the sum of $\frac{3}{80}$, $\frac{2}{7}$, $\frac{43}{56}$, $\frac{7}{24}$, $\frac{75}{436}$, in decimals, correct to fourth place.

41. The dividend is 9876, the quotient is 87, the remainder is 45. Find the divisor.

42. Change .03125 to a common fraction in smallest terms.

43. Bought a hogshead of sugar containing 848 pounds for $38.16, and paid $4.24 freight and cartage. At what price per pound must it be sold to gain 20%?

44. To $\frac{3}{5}$ of $\frac{3}{4}$ add $\frac{1}{2}$ of $\frac{7}{10}$, and reduce to lowest terms; multiply the sum so obtained by $1\frac{2}{3}$, and reduce to a mixed number; from the product subtract $\frac{5}{6}$, and reduce to lowest terms; divide the remainder by 5, and convert the quotient into a decimal fraction; add 1.1; multiply by 2.5; subtract .9; and divide the remainder by .007.

45. A can weigh a certain quantity of goods in 15 days by working 7 hours a day. How long will it take him to do the same work by working 9 hours a day?

46. In an example in division the remainder is 14, the divisor is 16, and the quotient is 18. What is the dividend?

47. Solve by cancellation:

How many pieces of cotton cloth, each piece containing 42 yards, at $9\frac{1}{2}¢$ per yard, can be bought for 14 firkins of butter, each containing 56 pounds, at 19¢ per pound?

48. What must be the depth of a bin which is 4 ft. wide and 6 ft. long, to contain 40 bushels oats?

49. A farmer sold 9875 pounds hay at $12\frac{1}{2}$ per ton, and took in part payment 5000 feet of boards at $11 per thousand. How much remained due him?

50. Bought 80 barrels of flour at $6 per barrel, paying for freight $30. At what price must I sell it per barrel to gain 30% on the total cost?

51. What is the amount of $720.50, for 3 yr. 5 mo. 19 da., at 6 per cent?

52. Three men buy a house for $2500. A pays $500, B pays $900, C pays $1100. They rent it for $250. What is each one's share of the rent?

53. If 12.875 acres of land cost $1030, what will 4.75 acres cost?

54. Write three-fourths of one per cent, first as a pure decimal, and again as a common fraction.

55. If a man paid $18\frac{3}{4}$ for a load of hay weighing $1\frac{4}{5}$ tons, what would he pay at the same rate for $\frac{2}{5}$ of a ton?

56. If 11 weavers in 9 days weave 1584 yards, what will 1 man do in 1 day? 6 men in 7 days?

57. What is the exact interest of $500, for 100 days, at 8 per cent? (Take 365 days to the year.)

58. Divide the product of $8\frac{3}{4}$ and $11\frac{2}{3}$ by their difference.

59. A merchant bought 340 bushels of potatoes at 80¢ per bushel; 20 per cent of them proved worthless, and were thrown away. He sold the remainder at $1.10 a bushel. What did he gain or lose?

60. Divide eighty-four and eighty-four hundredths by forty-eight thousandths.

61. How much money in silver dollars, $412\frac{1}{2}$ grains each, will weigh 165 pounds Avoirdupois, 7000 grains to the pound?

62. What is the amount of $1395, at 4 per cent, for 7 mo. 24 da.

63. A coal dealer buys 150 tons of coal, 2240 pounds each, at $4.50 per ton. He sells it at $4.75 per ton, giving 2000 pounds to the ton. What is his profit?

64. What is the value of $(\frac{2}{3}$ of $\frac{5}{6}$ of $3\frac{3}{4}+8\frac{2}{3}) \div (10\frac{1}{2}-7\frac{11}{12})$?

65. How many bushels of grain will fill a bin 8.5 feet long, 4.25 feet wide, and $3\frac{3}{4}$ feet deep?

66. Three workmen receive $\$283.50$ for doing a piece of work. One worked 32 days, the second worked 53 days, the third worked 41 days. What is the share of each?

67. A man bought silverware for $\$120$, and sold it for $\$250$ less $33\frac{1}{3}$ and 10 per cent. What was his profit per cent?

68. What is the interest on $\$356.75$, at 4 per cent, for 3 yr. 5 mo. 14 da.?

69. A note for $\$600$, drawn Jan. 16, payable 4 months after date, is discounted March 25 at a bank, at 6 per cent. What are the proceeds?

70. A dry-goods merchant sells goods $12\frac{1}{2}¢$ per yard more than their cost, and realizes a profit of 8 per cent. What is the cost per yard?

71. A man bought 396 acres of land for $\$40{,}293$. He sold 150 acres at $\$120$ per acre, 134 acres at $\$80$ per acre, and the remainder at cost. Did he gain or lose, and how much?

72. If $44\frac{2}{9}$ yards of calico cost $\$1.99$, how much must be paid for 80 yards?

73. Divide the sum of 75 thousandths and 75 ten-thousandths by the difference between 75 hundredths and 75 tenths.

74. What number divided by 320 gives 47 for quotient and 163 for remainder?

75. In a schoolroom there are 35 pupils and a teacher. The room is 30 feet long, 20 feet wide, and 15 feet high. How many cubic feet of air space has each person?

76. A merchant sold a quantity of flour for $\$282$, losing 6 per cent. How much money did he lose?

77. I bought 2500 bushels of wheat at 80¢ per bushel, and sold it for 84¢ per bushel, on a note for 60 days, which I had discounted immediately at a bank, at 6 %. How much did I gain?

78. A merchant bought 84 yards of linen at 55¢ per yard, and 105 yards of muslin at 20¢ per yard. He sold all the linen at 40¢ per yard. What must he charge per yard for the muslin in order to make up exactly his loss on the linen?

79. A fruit dealer bought a lot of oranges for $240. He sold $\frac{1}{4}$ of them for $\frac{1}{2}$ of the entire cost; $\frac{1}{3}$ of the remainder for $\frac{2}{5}$ of the entire cost; $\frac{1}{2}$ of what then remained for $\frac{1}{6}$ of the entire cost; and the final remainder for $\frac{1}{8}$ of the entire cost. What was his gain or loss?

80. The owner of 165 shares of gas stock sold them at $25 per share, and with the proceeds purchased two lots, 32 feet by 115 feet, and 30 feet by 105 feet, respectively, and had just $27 left. What was the price per square foot of the lots?

81. A man purchased a house, paying for it in four payments as follows: on the first payment $\frac{1}{3}$ of the purchase price; on the second payment $\frac{1}{2}$ of the remainder; on the third payment $\frac{3}{5}$ of what then remained due; and on the last payment $2000. What was the full amount paid for the house?

82. Find the difference between the greatest common divisor of 480 and 520, and the least common multiple of 5, 6, 15, and 20.

83. Find the value of a pile of wood 40 feet long, 8 feet wide, and 4 ft. 6 in. high, at $5.50 a cord.

84. A cargo of flour was bought for $690. For what must it be sold to gain $66\frac{2}{3}$ %?

85. Find the sum of all the prime numbers to 50.

86. If A and B can mow a field in seven days, and A, B, and C mow it in five days, for $25, what ought C to receive?

87. To $\frac{5}{6}$ of a score add $\frac{4}{5}$ of a dozen, and from the sum subtract $\frac{2}{9}$ of a hundred. What is the remainder?

88. What must be the length of a load of wood that is 4 feet wide and $5\frac{1}{3}$ feet high to contain 2 cords?

89. Bought a hogshead of molasses containing 128 gallons, at 65¢ a gallon; paid 80¢ for cartage, and lost 16 gallons by leakage. At what price per gallon must the remainder be sold to gain one-fifth of the entire cost?

90. What is the least number that will exactly contain 48, 20, 21, 24?

91. Sold 50 sofas for $2250. 25 of them were sold at a gain of 20 per cent, and 25 at a loss of 20 per cent. What was the gain or loss on the transaction?

92. Bought a number of eggs, and sold 11 of them for what 18 cost me. What was my gain per cent?

93. A bookseller wishes to mark up the price of a book which he is now selling for $2, so that he can deduct 15 per cent, and yet receive the present price. What must be the marked price?

94. What is the difference between .75 divided by 75, and 75 divided by .75?

95. A watch that loses 35 seconds in an hour was set right at noon on Monday. What time did it show at 6 P.M. the following Thursday?

96. Mr. A. sold a horse for $240, which was 20 per cent less than he asked for it, and his asking price was 20 per cent more than the horse cost him. What was the cost of the horse?

97. Three quarts dry measure is what per cent of a bushel?

98. What will it cost to carpet an office room measuring 21 feet in length, and 18 feet in width, the carpeting being $\frac{3}{4}$ yard wide, and costing $1.35 per lineal yard?

99. A physician accepts, in payment of a bill, a note for $275.75, due in one year and three months, with interest at 7 per cent. What amount will be due at maturity?

100. At what rate will $1500 amount to $1684.50, in 2 yr. 18 da.?

101. How shall I mark goods that cost me $.96 a yard, in order to abate 15% and still make 15%?

102. What will it cost to insure a factory valued at $21,000, at $\frac{4}{5}$%, and the machinery valued at $15,400, at $\frac{5}{8}$%?

103. In what time will $750 gain $195 interest, at 4%?

104. What is the rate per cent when the amount of $500 is $593.75, for 2 yr. and 6 mo.?

105. What principal will gain $360 in 5 yr. 4 mo., at $4\frac{1}{2}$%?

106. Bought 480 barrels of flour, at $4.50 a barrel, and sold it for $2880. Find the gain per cent.

107. By selling a house for $10,304, a man gained 15% on the cost. What was the cost?

108. A man, dying, left $\frac{2}{3}$ of his estate to his wife, $\frac{2}{3}$ of the remainder to his son, and the remainder to his daughter, who received $5000. What was the value of the estate, and what was the son's share?

109. What is the interest of $10, for 10 yr. 10 mo. 10 da., at 10 per cent?

110. If it takes one man $7\frac{1}{4}$ days to do a piece of work, how long will it take 3 men to do $2\frac{2}{3}$ times as much?

111. A grocer pays 18¢ per pound for coffee, and roasts it, losing 10% of the weight in the process. What must he charge per pound for the roasted coffee in order to make a profit of 20%?

112. A merchant bought 48 bales of cotton, and then sold the lot for $2008.80, losing 7%. What was the cost per bale?

113. What is the cost of sawing a pile of wood 20 feet long, 4 feet wide, and 6 feet high, at $1.20 a cord?

114. After increasing the wages of his workmen 33⅓%, a manufacturer paid them $2.60 a day. What did he pay them before?

115. What should a bookseller charge for a book for which he paid at the rate of $54 a dozen, that he may make 20% on the cost?

116. What is the per cent profit or loss when a hundred logs which cost $65 are sold at 78¢ each?

117. A man spent $\frac{8}{21}$, and invested in his business $\frac{4}{15}$, of his income. He deposited the remainder, $1850, in a bank. What was his income?

118. Sold a horse for $322, and thereby lost 8%. What should I have sold it for to gain 15%?

119. Bought a horse for $340; paid $60 for keeping him, and then sold him for $540. What per cent was gained?

120. John bought 12½ pounds of sugar at 5½¢ a pound, spending 25% of his money. How much had he at first?

121. When 10.25 bushels of wheat cost $12.71, what will 7½ bushels cost?

122. Mr. Jones paid $15.12 for the use of a sum of money for 1 yr. 6 mo., at 5%. What was the sum?

123. What were the proceeds of a note for $725.14, due July 7, discounted at a bank June 20, at 8%?

124. After Mr. Jones had spent $37\frac{1}{2}\%$ of his money, he found that he then had enough to buy 80 pounds of rice at $6\frac{1}{2}¢$ a pound. How much could he have bought with the whole of his money?

125. On the 10th day of November, 1899, you lent William Rogers $\$ 864.50$. How much does he owe you today, the rate of interest being $4\frac{1}{2}\%$?

126. A man bought wheat for $\$ 10{,}867$, and sold it at a gain of $4\frac{1}{2}\%$. What did he receive for it?

127. Divide three million by six thousand, and multiply the quotient by .024.

128. How much must I have invested at 5% that my income may be $\$ 2880$ per year?

129. Add these *across*, placing the totals in the space indicated; then add the totals:

						Totals.
14,305	10,702	18,346	37,946	43,865	17,387	
22,324	17,437	18,438	3,741	22,972	25,960	
13,849	67,431	34,965	12,674	32,905	1,468	
15,607	27,865	32,476	18,430	33,301	18,695	
19,898	13,460	27,686	23,492	13,852	26,973	

130. If 1998, or 27 per cent, of the inhabitants of a town are voters, how many inhabitants has the town?

131. Ten cows were sold for $\$ 690$, at a gain of 15 per cent. For how much per head on the average should they have been sold to gain 20 per cent?

132. Find the interest of $\$ 575.50$, for 1 yr. 10 mo. 15 da., at 5%.

CHAPTER VI.

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RATIO.

391. ***Ratio*** **is the relation which one number has to another of the same kind.**

The sign of ratio is the colon (:).

The ratio of 3 to 6 is expressed 3 : 6.

The colon (:) is the sign used in France and Germany to indicate division as well as ratio.

392. **The *terms* of the ratio are the numbers compared, the first being called the *antecedent*, and the second the *consequent*. Both terms constitute a *couplet*.**

The ratio of 3 to 6 is obtained by dividing the antecedent by the consequent; 3 : 6 means $\frac{3}{6}$, which is equal to $\frac{1}{2}$.

393. Oral Exercises.

Find the ratio of:

1. 175 to 700. $\frac{175}{700} = \frac{1}{4}$. *Ans.*

2. \$ 36.50 to \$ 18.25. $\frac{\$\ 36.50}{\$\ 18.25} = 2$. *Ans.*

NOTE. — The quotient is abstract.

3. 6 pecks to 5 bushels. $\frac{6 \text{ pecks}}{20 \text{ pecks}} = \frac{3}{10}$. *Ans.*

NOTE. — The antecedent and the consequent must be like numbers.

4. \$ 19 to \$ 95.
5. 20 mills to 1 dollar.
6. 7 tenths to 3 fifths.
7. 3 quarts to 4 gallons.
8. 1 gallon to 500 cubic inches.

394. Written Problems.

1. One line is 3 rd. 4 yd. long; the length of another is 5 rd. 1 ft. Find the ratio of the first to the second.

The antecedent 3 rd. 4 yd. is to be divided by the consequent 5 yd. 1 ft. As the divisor and the dividend must be like numbers, both terms of the couplet are reduced to feet. The division is indicated by writing the antecedent above the consequent as a fraction. The concrete fraction $\frac{61\frac{1}{2} \text{ ft.}}{83\frac{1}{2} \text{ ft.}}$ is changed to the abstract complex fraction $\frac{61\frac{1}{2}}{83\frac{1}{2}}$, which is reduced to a simple fraction by multiplying both terms by 2, giving $\frac{123}{167}$ for the result.

$$\frac{3 \text{ rd. } 4 \text{ yd.}}{5 \text{ rd. } 1 \text{ ft.}} = \frac{61\frac{1}{2} \text{ ft.}}{83\frac{1}{2} \text{ ft.}} = \frac{123}{167}. \quad \textit{Ans.}$$

Make the antecedent and the consequent like numbers, and divide the former by the latter.

2. M walks in 1 hr. 47 min. as far as N walks in 2 hr. 3 min. What is the ratio of M's speed to N's?

In this example is required the ratio of M's *speed* to N's. The antecedent is, therefore, M's speed, and the consequent is N's speed. As the distance walked is not given, x may be used to represent the number of feet or yards or miles walked by M in 107 minutes, and by N in 123 minutes. $\frac{x}{107}$ will represent the distance walked by M in 1 minute, or M's speed, and $\frac{x}{123}$, N's speed. The ratio of M's speed to N's will be $\frac{x}{107} \div \frac{x}{123}$, or $\frac{x}{107} \times \frac{123}{x}$. Cancelling x in each, the result is $\frac{123}{107}$, or $1\frac{16}{107}$. *Ans.*

3. One candle lasts 4 hr. 20 min.; another lasts 3 hr. 15 min. Find the ratio of the first to the second.

4. A pound of coffee costs $25\frac{7}{8}¢$; 1 pound of sugar costs $5\frac{3}{16}¢$. What is the ratio of price of sugar to that of coffee?

5. P earns in $19\frac{6}{7}$ days as much as Q in $18\frac{3}{4}$ days. What is the ratio of Q's daily earnings to P's? Of P's to Q's?

6. One wheel makes 600 revolutions in $8\frac{4}{7}$ seconds; a second makes 300 revolutions in $3\frac{1}{3}$ seconds. What is the ratio of the speed of the first wheel to that of the second?

7. The circumference of a circle is 12.5664 feet, and its radius is 2 feet. What is the ratio of the diameter to the circumference?

8. One train goes 40 miles in 50 minutes; another goes 24 miles in a half hour. What is the ratio of the speed of the second to that of the first?

Find the number of miles each goes in an hour.

9. One window is 6 ft. 8 in. by 4 ft. 2 in.; a second is 4 ft. 3 in. by 2 ft. 1 in. What is the ratio of the area of the second to that of the first?

$$(4\tfrac{2}{3} \times 2\tfrac{1}{12}) \div (6\tfrac{2}{3} \times 4\tfrac{1}{3})$$

10. A mother is now 35 years old, and her son is 3 years and 6 months old. Fourteen months ago what was the ratio of the mother's age to that of her son?

11. A farm costing $ 4750 was sold for $ 5750. What is the ratio between the profit and the cost?

12. A man can do a piece of work in $4\frac{2}{7}$ days. What part of it can he do in a day and a half? What decimal? What per cent?

13. What is the ratio between a ton of 2000 pounds and one of 2240 pounds?

395. Oral Problems.

1. One line is a rod long; another is $5\frac{1}{2}$ ft. long. What is the ratio of the first to the second?

2. What is the ratio of 7 hours to one day?

3. A pound of coffee costs 30¢, of sugar 6¢. What is the ratio of their respective prices?

4. A walks in 4 hours as far as B in 5. What is the ratio of A's speed to B's?

5. E earns in 6 days as much as D earns in 8 days. Find the ratio of E's daily earnings to D's.

6. One wheel makes 300 revolutions in 2 minutes; the second requires only $1\frac{1}{2}$ minutes to make the same number. Find the ratio of the number of revolutions made by the first wheel in 1 minute to the number made by the second wheel in the same time.

7. A circle whose diameter is 1 foot has a circumference of $3\frac{1}{7}$ feet. What is the ratio of the diameter to the circumference?

8. One train goes 40 miles an hour; a second goes 45 miles an hour. What is the ratio of the speed of the first to that of the second?

PROPORTION.

396. Preliminary Exercises.

1. $\frac{3}{16} = \frac{?}{64}$.
2. $\frac{18}{37} = \frac{36}{?}$.
3. $\frac{15}{13} = \frac{?}{65}$.
4. $\frac{1 \text{ pk.}}{3 \text{ bu.}} = \frac{\$\,?}{\$\,24}$.
5. $\frac{3 \text{ qt.}}{1 \text{ gal.}} = \frac{30¢}{?\,¢}$.
6. $\frac{17}{21} = \frac{51}{?}$.
7. $\frac{18}{?} = \frac{36}{70}$.
8. $\frac{?}{24} = \frac{57}{72}$.
9. $\frac{\$\,16}{?} = \frac{7 \text{ marks}}{21 \text{ marks}}$.
10. $5 \div 22 = ? \div 88$.
11. 6 horses $\div$? horses $= \$\,600 \div \$\,900$.
12. 1 ft. $\div$? yd. $= 15¢ \div 90¢$.
13. 1 qt. 1 pt. $\div$ 1 pt. $= ?¢ \div 4¢$
14. $1\frac{1}{4} \div \frac{3}{4} = \frac{?}{8} \div \frac{6}{8}$.
15. $2.8 \div .4 = .14 \div x$.

397. Two equal ratios form a proportion.

The ratio of 3 to 9 is $\frac{1}{3}$, which is also the ratio of 13 to 39. This may be expressed $\frac{3}{9} = \frac{13}{39}$, or $3 : 9 = 13 : 39$. Substituting a double colon (: :) for the sign of equality (=), we have the following proportion:

$$3 : 9 : : 13 : 39.$$

This is read, 3 is to 9 as 13 is to 39.

In the foregoing proportion, 3 and 13 are the *antecedents*, and 9 and 39 are the *consequents*.

398. **The first and the last term of a proportion constitute the *extremes*; the second and the third the *means*.**

In the following proportion

$$5 : 15 : : 9 : 27$$

5 and 27 are the extremes, 15 and 9 are the means.

The foregoing proportion may be written

$$\tfrac{5}{15} = \tfrac{9}{27}.$$

Multiplying each of these two fractions by the product of the denominators, 15×27, we have

$$\frac{5 \times 15 \times 27}{15} = \frac{9 \times 15 \times 27}{27}.$$

Cancelling, $5 \times 27 = 9 \times 15.$

In the same way it may be shown that in any proportion the product of the numbers in the extremes is equal to the product of the numbers in the means.

399. Written Exercises.

Find the missing term.

1. $3 : 4\tfrac{4}{5} :: 5 : x.$

As the product of the extremes is equal to the product of the means, 3 multiplied by x is equal to $4\frac{4}{5}$ multiplied by 5; *i.e.* $3x = 4\frac{4}{5} \times 5$. x, therefore, is equal to $\frac{4\frac{4}{5} \times 5}{3}$. This reduces to $\frac{24}{3}$, or 8. *Ans.*

To find an extreme, divide the product of the means by the other extreme.

2. $\frac{5}{9} : \frac{11}{21} :: x : \frac{11}{24}.$

The product of the means $\frac{11}{21} \times x$ equals the product of the extremes $\frac{5}{9} \times \frac{11}{24}$. x is equal, therefore, to $\frac{5}{9} \times \frac{11}{24} \div \frac{11}{21}$. Inverting the divisor, we have $\frac{5}{9} \times \frac{11}{24} \times \frac{21}{11}$. Cancel.

To find a mean, divide the product of the extremes by the other mean.

3. $3\frac{1}{2} \div 16 = \frac{7}{8} \div x.$

4. $5 : 7 :: 12\frac{1}{2} : x.$

5. $3 \div x = 12 \div 20.$

6. $? : 19 :: 28 : 76.$

7. $x : 15 :: 4 : \frac{8}{7}.$

8. $x : \frac{1}{2} :: 2 : 7.$

9. $\frac{2}{3} : x :: \frac{1}{4} : \frac{3}{5}$.

10. $\frac{1}{9} : \frac{8}{9} :: x : 2\frac{1}{4}$.

11. $1 : \frac{5}{8} :: 1\frac{3}{5} : x$.

12. $x : \frac{5}{9} :: 11 : 3\frac{1}{3}$.

13. $x : 9 :: 4 : x$.

14. 1 lb. 1 oz. : 2 lb. 4 oz. : : 17¢ : x¢.

15. 3 qt. 1 pt. ÷ 1 gal. = x¢ ÷ 80¢.

16. 4 bottles : x bottles = 6 pints : 15 pints.

17. x men : 9 men = 16 acres : 36 acres.

400. Oral Problems.

1. If 9 eggs cost 25¢, what will 3 dozen cost?

EXPLANATION. — 3 dozen, or 36, will cost 4 times as much as 9; 4 times 25¢ = $1.

2. If 7 pounds of flour cost 23¢, what will be paid for 49 pounds?

3. For $5 I can get 12 straw hats. How many can I get for $20?

4. A wheel makes 75 revolutions in 5 minutes. How many does it make in an hour?

5. $100 principal gives $6 interest. How much will be the interest of $450 principal?

6. A merchant pays 75¢ freight on 125 pounds of merchandise. How much will be the freight on 1000 pounds at the same rate?

7. A locomotive goes 3 miles in 4 minutes. How far does it go in an hour?

8. 4 horses can eat a certain quantity of hay in 10 months. How long will it last 20 horses?

9. 12 men can do a piece of work in 15 days. How long will 36 men require?

10. 15 yards cost 270 cents. What will be the cost of 5 yards?

401. Written Problems.

1. If 9 cows cost $267, what will be the cost of 36 at the same rate?

The ratio of the cost, $267 : x, must be the same as the ratio of the number of cows, 9 : 36. Making the proportion, we have

$$9 : 36 :: 267 : x.$$

Therefore, $$x = \frac{\$267 \times 36}{9}.$$

Cancelling, $$x = \$1068. \textit{Ans.}$$

2. 7 barrels of sugar cost $104.32. Find the cost of 42 barrels at the same rate.

3. A wheel makes 248 revolutions in 5 minutes. How many does it make in 1 hour 20 minutes?

Make the required number of revolutions the fourth term. The proportion will then be as follows:

5 minutes : 80 minutes : : 248 revolutions : x revolutions.

$$x = \frac{248 \text{ revolutions} \times 80}{5}.$$

4. A locomotive goes 2.8 miles in 4 minutes. How far does it go in an hour?

5. From 9 pounds of yarn are made 42 yards of dress goods. How many yards can be made from 165 pounds of yarn?

How many pounds of yarn are needed for 196 yards of goods?

6. If 17 men receive $357 for a week's work, how much should 24 men receive?

7. If 17 men take 27 days to finish some work, how long would it take 51 men?

NOTE. — The *work* done by 51 men would be $\frac{51}{17}$ of the work done by 17 men. The *time* required by 51 men would be $\frac{17}{51}$ of the time required by 17 men.

8. When a sum of money is divided among 48 persons, each receives \$ 27.50. How much would each receive if the same sum were divided among 16 persons?

9. For \$ 85 I can purchase 238 yards of dress goods. How many yards can I purchase for \$ 5?

10. A can do a piece of work in 6 days; B can do it in 7 days. If B's wages are \$ 2.10 per day, how much should A receive per day?

11. If for 7*s.* 6*d.* I can buy 9 pounds of raisins, how many pounds can I buy for £ 56 16*s.*?

12. A quantity of provisions would last a ship's crew 20 days, allowing each man 2 lb. 4 oz. daily. What should each man be allowed so as to make the provisions last 4 days longer?

24 days : 20 days : : 36 ounces : x ounces.

13. If 40 men are able to do a piece of work in 10 hours, how many extra men must be employed to finish it in 8 hours?

8 hours : 10 hours : : 40 men : x men. The number of extra men is $x - 40$.

14. If it requires 40 yards of carpet 2 ft. 9 in. wide to cover a floor, how many yards of carpet 2 ft. 6 in. wide would be needed?

15. How long will it take a train to go 112 miles, at the rate of 46 miles in 1 hr. 20 min. 30 sec.?

16. If a beam 5 ft. 6 in. long, 10 inches wide, and 8 inches thick weighs 924 pounds, find the length of another beam of the same material which weighs 3024 pounds, and whose end is a square foot.

PARTITIVE PROPORTION.

402. Preliminary Exercises.

1. Coin silver consists of 9 parts silver and 1 part copper. What is the ratio of the weight of the silver in a dime to the weight of the coin?

2. What is the ratio of the weight of the copper to the weight of the coin?

3. How many ounces of copper are there in a bar of coin silver weighing 90 ounces? How many ounces of pure silver?

403. *Partitive proportion* is the process of dividing a number into parts proportional to given numbers.

404. Written Problems.

1. Divide 180 into parts proportional to 2, 3, and 4.

2 / 3 / 4

If the parts were 2, 3, and 4, the whole number would be 2 + 3 + 4, or 9. The ratio of the whole to the first part must be 9 to 2; of the second, 9 to 3; of the third, 9 to 4. These ratios give rise to the proportions indicated.

$9 : 2 :: 180 : x.$ $\therefore x = 40.$
$9 : 3 :: 180 : y.$ $\therefore y = 60.$
$9 : 4 :: 180 : z.$ $\therefore z = 80.$ *Ans.* 40, 60, and 80.

2. Gunpowder is composed of 15 parts of saltpeter, 2 of sulphur, and 3 of charcoal, mixed together. How many pounds of each are there in 72 pounds of powder?

15 / 2 / 3

In a mixture of 15 lb. + 2 lb. + 3 lb., or 20 lb., there will be 15 lb. saltpeter; hence, the ratio of the whole weight to the weight of the saltpeter is 20 lb. to 15 lb., etc.

20 : 15 :: 72 : number of pounds of saltpeter.
20 : 2 :: 72 : number of pounds of sulphur.
20 : 3 :: 72 : number of pounds of charcoal.

3. A bankrupt surrenders property worth $1287 for the benefit of three creditors to whom he owes $750, $1125, and $1245, respectively. How much should each creditor receive?

4. A had on storage in a warehouse 2400 bales of cotton, B 1500 bales, and C 1100 bales. After a fire that destroyed all distinguishing marks, the damaged cotton was sold for $10,000. How should this sum be divided?

5. Our standard gold coin consists of 900 parts gold, 90 parts silver, 10 parts copper. What is the quantity of each metal in 50 pounds of coin?

6. Two men hire a pasture for $45. One puts in 15 cows; the other puts in 12 cows. What should each pay?

7. A and B hire a boat for 50 days, paying $30. A uses it 27 days; B uses it 23 days. How much should each pay?

8. Three farmers together paid $54 for threshing their grain. A threshed his crop of 900 bu.; B threshed his crop of 828 bu.; C 672 bu. What did each pay?

9. A and B contract to haul a pile of lumber for $105. A furnishes 3 teams, and B 4 teams. How much does B receive?

10. Three merchants shipped a cargo of iron by sea. A sent 180 tons, B sent 105 tons, C sent 315 tons. During a storm the sailors were obliged to throw overboard 180 tons to save the vessel. Assuming that the cargo should sustain one fourth of the loss, what portion of the loss should each merchant sustain?

11. Divide 90 into two parts which shall be to each other as 9 to 1.

PARTNERSHIP.

405. Written Problems.

1. B and C gain by trade $182. What is the gain of each, B having put in $300, and C $400?

The total investment is $700. The ratio of the total investment to B's investment is 700 to 300. This should be the ratio of the total profit to B's share, etc.

300
400
700 : 300 : : $182 : B's share.
700 : 400 : : $182 : C's share.

Make proportions whose antecedents in each case are the total investment and the total profit, the consequents being the investment of one partner and his share of the profit.

2. A, B, and C invest $720, $340, and $960, respectively. The profits are $101. What is each one's share?

3. A, B, and C buy a house for $7500. A furnishes $2000; B, $2500; C, the remainder. The yearly rent, less expenses, is $576. To what amount is each entitled?

4. M and N entered into partnership. M puts $200 into the business for 5 months, and N $300 for 4 months. They gained $132. Find the share of each.

An investment of $200 for 5 months is equivalent to an investment of $1000 for 1 month; an investment of $300 for 4 months, to $1200 for 1 month.

$200 \times 5 = 1000$
$300 \times 4 = 1200$
2200 : 1000 : : $132 : M's share.
2200 : 1200 : : $132 : N's share.

In ascertaining the ratio of the whole capital to the share contributed by each, $1000 and $1200 are taken as representing the shares of each in a total capital of $2200.

Multiply each partner's share of the capital by the time it is in the business, and consider the products, respectively, as the sums contributed by the partners.

NOTE. — This mode of ascertaining a partner's share of profits or losses is based upon the assumption that the agreement of the partners does not provide for a different division.

5. X and Y rent a field for $ 32. X puts in 8 horses for 6 months, and Y 10 horses for 8 months. How many dollars should each pay?

8 horses for 6 months = how many for one month?

10 horses for 8 months = how many for one month?

6. Three men hire a pasture for $ 84. One puts in 15 cows for 12 weeks; the second puts in 20 cows for 6 weeks; the third puts in 18 cows for 10 weeks. What amount should each pay?

7. Four men hire a pasture field together. The first pastures 4 cows 18 weeks; the second, 5 cows $12\frac{4}{7}$ weeks; the third, 11 cows $6\frac{1}{2}$ weeks; the fourth, 9 cows 16 weeks. What part of the rent should each pay?

8. Two men hire a pasture for $ 420. A puts in 300 sheep for 5 weeks, and B puts in 450 sheep for 6 weeks. What should each pay?

9. A, B, and C enter into partnership. A puts in $ 500 for 4 months, B $ 400 for 6 months, and C $ 800 for 3 months; they gain $ 340. Find each man's share of the gain.

10. A partnership is formed between A with a capital of $ 1500 and B with a capital of $ 2500. Six months thereafter they take in C with a capital of $ 4000. How should a profit of $ 3500 be divided at the end of the year?

11. A and B form a partnership. A furnishes $ 2000, B $ 3000. After a year A furnishes an additional $ 1000. At the end of 2 years the business is disposed of for $ 7100. How much should each receive?

Suggestion. — A receives his $ 3000 and how much of the profits? Should he receive as much as B, who had $ 3000 in the business the whole time?

COMPOUND PROPORTION.

406. A ***compound proportion*** **is one in which either ratio is compound.**

407. Written Problems.

1. If 72 men dig a ditch 20 yd. long, 1 ft. 6 in. broad, 4 ft. deep, in 3 days of 10 hours each, how many men would be required to dig a ditch 30 yd. long, 2 ft. 3 in. broad, and 5 ft. deep, in 15 days of 9 hours each?

Since the number of men is required, 72 men is made the third term of the proportion. Considering the length alone, the ratio of 72 men to the required number would be equal to the ratio of 20 feet to 30 feet. Considering the width, the ratio would be 18 inches to 27 inches. Considering the depth, the ratio would be 4 feet to 5 feet. Considering the number of days, the ratio would be 15 days to 3 days. Considering the number of hours per day, the ratio would be 9 hours to 10 hours. Dividing the product of the means by the product of the extremes, the number of men is found to be 45.

$$\begin{array}{l} 20 : 30 \\ 18 : 27 \\ 4 : 5 :: 72 : x \\ 15 : 3 \\ 9 : 10 \end{array} \qquad \begin{matrix} \\ \\ \text{men} \quad \text{men} \\ \\ \\ \end{matrix}$$

$$x^{\text{men}} = \frac{72 \times 30 \times 27 \times 5 \times 3 \times 10}{20 \times 18 \times 4 \times 15 \times 9}$$

Place the number required as the fourth term, making the like number the third term. Arrange the couplets, considering the effect of each separately on the result. Divide the product of the means by the product of the extremes.

2. If 45 horses eat $1\frac{1}{2}$ tons of hay in 30 days, how many tons should last 84 horses 56 days?

3. If 4 men, working 8 hours per day, can mow a meadow in 3 days, how many men, working 9 hours per day, can mow a meadow three times as large in 4 days?

4. If 10 men, working 8 hours per day, can build a certain wall in 6 days, how many hours a day must 12 men work to build the same wall in 4 days?

5. If 108 men can build a fort in $12\frac{1}{2}$ days of $12\frac{1}{2}$ hours each, in how many days can 84 men build it by working $10\frac{1}{2}$ hours daily?

6. What will it cost to transport 1000 pounds of mail matter 1000 miles, at $1 per 100 pounds per 100 miles?

7. If 12 men can do a piece of work in 20 days, what number of men will be required to do four times as much work in a fifth part of the time?

8. If 14 men can mow 168 acres in 12 days of 8 hr. 15 min. each, how many acres can 20 men mow in 11 days of 7 hr. 48 min. each?

9. If 5 needlewomen can do a piece of work in 11 days of 9 hours each, how long will it take 3 needlewomen to do two such pieces, supposing them to work $10\frac{1}{2}$ hours each day?

10. A employs a capital of $2500 in business, and at the end of 3 years takes into partnership B, who furnishes $4000. Four years later they are joined by C, with a capital of $5000. At the end of 12 years from the commencement of the business the profits, amounting to $15,000, are divided. What amount should each receive?

A's money is in the business how many years? B's how many years? C's how many?

11. A and B rented a field for a year for $175. A put in 6 horses for the whole time; B put in 5 horses for 11 months and 3 horses for 5 months. How much of the rent had each to pay?

12. A field of grain was to be cut down by 40 men in 10 days. Eight of the men, however, failed to come. How long did it take the others to do the work?

REVIEW.

408. Oral Problems.

1. How many weeks will $4\frac{1}{5}$ tons of coal last Mrs. Bright, if she uses $\frac{3}{10}$ of a ton each week?

2. I can buy 2 pairs of shoes for 12 shillings. How many pairs at the same rate can I buy for £ 3?

3. If two-thirds of your age is 8 years and 4 months, how old are you?

4. 5 quarts equal what decimal of a peck?

5. What is the cost of 700 pounds of coal at $ 7 a ton?

6. How much would you pay for $2\frac{2}{3}$ yards of cloth at $37\frac{1}{2}$¢ a yard?

7. In what time will $ 50, at 6%, give $ 18 interest?

8. If I buy an article for $ 75 and sell it for $ 50, what is my loss per cent?

9. By selling an article for $ 9, a man gained 25%. How many dollars would he have gained if he had sold the article at an advance of 50% over cost?

10. How many quarts of peanuts in 1 bushel and 3 pecks?

11. What would be the cost of 120 books at $66\frac{2}{3}$¢ each?

12. Change 66,321 mills to dollars.

13. $ 120 is $\frac{4}{5}$ per cent of what number of dollars?

14. In what time will $ 50 double itself at 8%?

15. If $ 1 is paid for insuring a piano worth $ 500, what is the rate of insurance?

16. Into how many lots, containing $\frac{2}{3}$ of an acre each, can 8 acres be divided?

17. A man lends $ 1200 at 6%, and 1500 at 5%. What is the difference in the amount of yearly interest due on each?

18. A man owning $\frac{2}{3}$ of a ship sold $\frac{2}{3}$ of his share. What part of the ship did he still own?

19. How many rings, each 2 pwt. 12 gr., can be made from $\frac{1}{4}$ pound of gold?

20. Find the number of square inches on the surface of a block 10 inches long by 4 inches wide by 3 inches thick.

409. Written Problems.

1. A traveller walked $23\frac{1}{8}$ miles the first day, $3\frac{3}{4}$ miles more the second day than the first, and $3\frac{1}{8}$ miles more the third day than the second. How far did he walk in the three days?

2. Multiply 63.15 by 1.04; divide the product by 6.25, and subtract the quotient from 11

3. How many bricks, 8 inches long and 4 inches wide, will be needed to make a sidewalk 26 feet long and 4 feet wide?

4. If it costs \$10.24 to carry 1500 pounds 356 miles, what will it cost to carry 2700 pounds 890 miles?

5. A house rents for \$30 a month, and the owner pays \$75 a year for taxes and repairs. What is the value of the house, if his net profit is 5 per cent per annum?

6. A loaned B a sum of money at $4\frac{1}{2}$ per cent interest per annum. At the end of 18 months B paid the debt, principal and interest, in all \$1814.75. What was the sum borrowed?

7. If a 5-months note for \$760, dated March 13, is discounted at a bank May 23, the rate being 7 per cent a year, what will be the proceeds?

8. A grocer bought 40 gallons of maple syrup at the rate of 4 gallons for \$6, and sold it at the rate of 5 gallons for \$8. What was the whole gain, and the gain per cent?

9. Two pictures were sold for $99 each. On one there was a gain of 10%; on the other a loss of 10%. Was there a gain or a loss on the sale of both, and how much?

10. New York, Jan. 1, 1904.

One year after date I promise to pay J. Edward Swanston Eight Hundred Dollars for value received, with interest.

800\frac{00}{100}$. Rufus L. Scott.

Indorsed as follows: Apr. 1, 1904, $10; July 1, 1904, $35; Nov. 1, 1904, $100.

What was due Jan. 1, 1905? (Merchant's Rule.)

11. What is the difference on a bill of $780, between a discount of 40% and a discount of 35 and 5%?

12. How many cords in a pile of wood 42 feet long, 12 feet high, and 8 feet wide? Find its cost at $6.35 per cord.

13. What principal, on interest for 2 yr. 6 mo. at 4%, will gain $850?

14. What is the cost of insuring a house, worth $25,000, for $\frac{3}{4}$ of its value at $1\frac{1}{2}$%?

15. At 9¢ a cubic foot what will be the cost of a block of stone 9 ft. long, 4 ft. wide, and 5 ft. 6 in. thick?

16. If a steeple 150 feet high casts a shadow of 275 feet, how long a shadow will be cast by a man 6 feet tall, at the same time of day?

17. The tax to be raised in a certain town is $1350. The taxable property is valued at $108,000 What is the tax on one dollar?

18. Mr. Fox buys one-fifth of an acre of land for $21.78. For how much a square foot must he sell it to gain 20%?

19. What is the cost of covering a floor $16\frac{1}{2}$ ft. long, 12 ft. wide, with oil-cloth $1\frac{1}{2}$ yd. wide, at 75¢ a yard?

20. The edges of a large cubical box are 5 feet long. How many square feet of paper will cover the outside of the box?

21. A field 110 yards long and 44 yards wide contains an acre. What is the area of a field 220 yards long and 88 yards wide? Of one 440 yards long and 176 yards wide?

$$110 \times 44 : 220 \times 88 :: 1 \text{ acre} : x \text{ acres.}$$

22. If a steel bar 12 feet long, 4 inches broad, and $2\frac{1}{2}$ inches thick weighs 480 pounds, what is the weight of another steel bar 18 feet long, 3 inches broad, and 2 inches thick?

23. At a certain hour a pole 6 feet high casts a shadow measuring 4 ft. 2 in. Calculate the height of a steeple whose shadow at the same hour is 104 ft. 2 in.

24. If 7 men receive \$126 for 5 weeks' work, how much should they receive for 9 weeks' work?

25. If 76 boards, each 14 feet long and 10 inches wide, are worth \$19.76, how much would 50 such boards be worth?

INVOLUTION.

410. ***Involution*** **is finding any power of a number.**

A *power* of a number is the product obtained by using the number a certain number of times as a factor.

2 is the first power of 2. 2×2, or 4, is the second power of 2. $2 \times 2 \times 2$, or 8, is the third power of 2, etc.

411. The second power of a number is called its *square*; the third power is called its *cube*.

412. The power of a number is indicated by writing a small figure, called an *exponent* a little to the right of the upper part of a number.

The square of 2 is written 2^2.
The cube of 2 is written 2^3.
The fourth power of 2 is written 2^4.

$$5^2 = 25, \quad 12^2 = 144.$$

What is the square of 4? Of 6? Of 7? Of 9? Of 10? Of 11?

Square 13. 15. 21. 16. 19. $14^2 = ?$ $17^2 = ?$ $54^2 = ?$ $33^2 = ?$

413. The square of $25 = (20 + 5) \times (20 + 5)$.

$$\begin{array}{ll} & 20 + 5 \\ & 20 + 5 \\ \hline \text{Multiplying by } 20 & 20^2 + \quad 20 \times 5 \\ \text{Multiplying by } 5 & \qquad\quad 20 \times 5 \; + 5^2 \\ \hline & 20^2 + 2(20 \times 5) + 5^2 = 400 + 200 + 25 = 625. \end{array}$$

414. The square of the sum of two numbers is equal to the square of the first + twice the product of the first by the second + the square of the second.

$$13^2 = (10 + 3)^2 = \; 10^2 + 2(10 \times 3) + 3^2 = ?$$
$$18^2 = (10 + 8)^2 = 100 \; + 160 + 64 = ?$$
$$27^2 = (20 + 7)^2 = 400 \; + 280 + 49 = ?$$

415. Oral Exercises.

Square:

1. 19.	**4.** 26.	**7.** 51.	**10.** 32.	**13.** 27.
2. 22.	**5.** 31.	**8.** 61.	**11.** 42.	**14.** 33.
3. 24.	**6.** 41.	**9.** 23.	**12.** 52.	**15.** 43.

EVOLUTION.

416. ***Evolution*** **is finding any root of a number.**
A *root* is one of the equal factors of a number.
The *square root* of a number is one of its two equal factors.

The square root of 4 is 2; of 9 is 3; of 16 is 4; of 25 is **5.**

417. Give the square root of 36. Of 64. Of 81. Of 121. Of 49. Of 100. Of 144.

418. The sign of a square root is $\sqrt{\ }$.

$\sqrt{81} = 9.$ $\sqrt{121} = ?.$ $\sqrt{25} = ?.$ $\sqrt{49} = ?.$

SQUARE ROOT.

419. Find the square root of 169.

$10^2 = 100.$ $20^2 = 400.$ The square root is between **10 and 20; it is,** therefore, 10 + a second number.

$$169 = 10^2 + 2\,(10 \times \text{second}) + \text{second}^2.$$

$$169 = 100 + 20 \times \text{second} + \text{second}^2.$$

$$20 \times \text{second} + \text{second}^2 = 69.$$

From this it appears that the second number is 3, since

$$20 \times 3 + 3^2 = 69.$$

420. It may be shown in this way:

$$\begin{array}{rl} & 10 \text{ (first number)} \\ & \overline{169} \\ 10^2 = & 100 \\ \hline \text{Trial divisor — twice 10} \quad 20) & 69 (3 \text{ second number}) \\ & 60 \\ & \overline{9} \\ 3^2 = & 9 \end{array}$$

Ans. $10 + 3 = 13.$

421. Find the square root of 2116.

$$
\begin{array}{rl}
 & \underline{40}\ \text{(first number)} \\
 & 2116 \\
40^2 & \underline{1600} \\
40 \times 2 = 80, \text{ trial divisor } &)516(6\ \text{second number}) \\
 & \underline{480} \\
 & 36 = 6^2
\end{array}
$$

Ans. 46.

Instead of multiplying the trial divisor by the second number, and then ascertaining whether the remainder is the square of the second number, the second number is added to the trial divisor and this sum is multiplied by the second number.

In practice, the work is shortened by omitting the ciphers.

$$
\begin{array}{rl}
 & \underline{40}\ \text{(first number)} \\
 & 2116 \\
 & \underline{1600} \\
(2 \times 40) + 6 = 86) & 516(6\ \text{second number}) \\
 & \underline{516}
\end{array}
\qquad
\begin{array}{rl}
 & \underline{4\ \ 6}\ \textit{Ans.} \\
 & 21'16 \\
 & \underline{16} \\
86) & 516 \\
 & \underline{516}
\end{array}
$$

First, point off in periods of two figures, commencing at units. Find the greatest square in the first period, and place the root in the quotient. Subtract the square from the first period and bring down the next period. Multiply the quotient figure by 2, and use it as a trial divisor. Place the second figure in the quotient and annex it also to the trial divisor. Multiply the figures in the trial divisor by the second quotient figure. Bring down the next period, and proceed as before until the square root is found.

422. Written Exercises.

Extract the square root:

1. 196.	**4.** 1225.	**7.** 2809.	**10.** 6889.
2. 324.	**5.** 1764.	**8.** 3721.	**11.** 8281.
3. 676.	**6.** 1936.	**9.** 5184.	**12.** 9025.

423. Find the square root:

NOTE. — Extract the square root of each term separately.

1. $\frac{4}{25}$.
2. $\frac{9}{64}$.
3. $\frac{169}{441}$.
4. $\frac{36}{121}$.
5. $\frac{49}{144}$.
6. $\frac{225}{1024}$.
7. $\frac{324}{2209}$.
8. $\frac{529}{8844}$.
9. $\frac{5929}{9025}$.

NOTE. — Before extracting the square root of the following, reduce the mixed numbers to improper fractions.

10. $12\frac{1}{4}$.
11. $11\frac{1}{9}$.
12. $2\frac{46}{49}$.
13. $10\frac{311}{625}$.
14. $156\frac{1}{4}$.
15. $264\frac{1}{16}$.

424. Find the square root of 425,104.

	6 5 2
	42'51'04
12|5	6 51
130|2	26 04 *Ans.* 652.

In finding any figure of the root after the first, we multiply the other figure or figures by 2 for a trial divisor.

425. Find the square root of 20,857,489.

	4 5 6 7
	20'85'74'89
8|5	4 85
90|6	60 74
912|7	6 38 89 *Ans.* 4567.

Find the square root of

1. 64,516.
2. 73,441.
3. 18,769.
4. 71,824.
5. 141,376.
6. .702244.

426. Written Exercises.

Find the square roots to two decimal places:

1. 7. **2.** 14. **3.** 38. **4.** 74. **5.** 350.

6. Find the square root of 3.6.

$\sqrt{3.6} = \sqrt{3.60}$

NOTE. — Commence at the units and point off two places to the right as well as to the left, annexing a decimal cipher, if necessary.

$$\begin{array}{r|l} & 1.89+ \\ & \overline{3.60'00} \\ 1 & \\ \hline 2.8 & 2.60 \\ & 2.24 \\ \hline 3.69 & .3600 \\ & .3321 \end{array}$$

7. 6.4. **8.** .121. **9.** .144. **10.** .196. **11.** .225

APPLICATIONS OF SQUARE ROOT.

427. Written Problems.

1. How many inches in the side of a square table top containing 529 square inches?

2. The surface of a square piece of board contains 3 sq. ft. 97 sq. in. What is the length of one side in feet and inches? (Reduce area to square inches.)

3. How many rods long is a square field containing 90 acres? How many yards of fence would be needed to enclose it?

4. Land surveyors use a measure called a *chain*. What is its length in rods, 10 square chains being equal to an acre? What is the length in feet?

It is subdivided into 100 "links." Find the length of a link in inches and decimal.

5. The surface of the six equal faces of a cube is 1350 square inches. What is the length of each edge of the cube?

428. Preliminary Exercises.

1. Carefully construct a right-angled triangle, base, 4 inches, perpendicular, 3 inches. Measure the hypotenuse.

Take the square of the length of each side and endeavor to show the relation between the square of the hypotenuse and the squares of the other two sides.

2. Construct a right-angled triangle, base, 3 inches, perpendicular, $1\frac{1}{4}$ inches. Measure the hypotenuse, and see if the relation between this hypotenuse and the other two sides of this triangle is the same as that found in the other triangle.

3. A right-angled triangle has a base 12 inches long; its perpendicular is $3\frac{1}{2}$ inches. What is the length of the hypotenuse?

4. The hypotenuse of a right-angled triangle is 25 inches; its perpendicular is 7 inches. What is the base?

5. The base of a right-angled triangle is 12 feet; the hypotenuse is 13 feet. Find the perpendicular.

429. Draw a right-angled triangle (Fig. 1). Upon each side construct a square (Fig. 2). From the upper portion of the largest square *C*, cut a right-angled triangle of the same

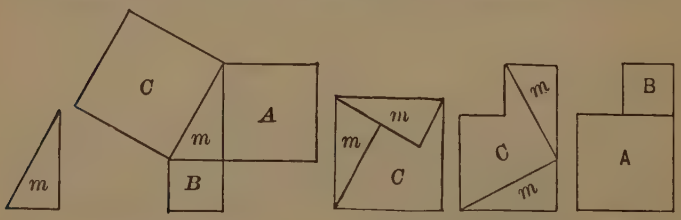


Fig. 1 Fig. 2 Fig. 3 Fig. 4 Fig. 5

dimensions as those of the original triangle *m*. Cut another triangle of the same dimensions from the left-hand portion (Fig. 3). Place one of these triangles below the remainder

of the square C, and the other at the right, as in Fig. 4, and the resulting polygon will be exactly equal in surface to the two squares A and B (Fig. 5).

430. The square described on the hypotenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.

431. Written Exercises.

Find the missing side of each of the following ten right-angled triangles:

1. Base, 15; perpendicular, 8; hypotenuse, ?.

$$\text{Square of hypotenuse} = 15^2 + 8^2$$
$$= 225 + 64$$
$$= 289.$$
$$\text{Hypotenuse} = \sqrt{289}, \text{ or } 17.$$

To find the hypotenuse, extract the square root of the sum of the squares of the other sides.

2. Base, 35; perpendicular, ?; hypotenuse, 37.

$$B^2 + P^2 = H^2;$$
$$35^2 + P^2 = 37^2;$$
$$1225 + P^2 = 1369;$$
$$P^2 = 144;$$
$$P = 12.$$

To find the base or the perpendicular, extract the square root of the square of the hypotenuse diminished by the square of the given side.

3. Base, ?; perpendicular, 15; hypotenuse, 39.

4. Base, 20; perpendicular, 21; hypotenuse, ?.

5. Base, ?; perpendicular, 45; hypotenuse, 53.

6. Base, 56; perpendicular, ?; hypotenuse, 65.

7. Base, 55; perpendicular, 48; hypotenuse, ?.

8. Base, ?; perpendicular, 14; hypotenuse, 50.

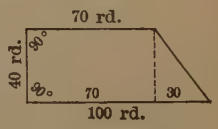
9. Base, 63; perpendicular, ?; hypotenuse, 65.

10. Base, 112; perpendicular, 15; hypotenuse, ?.

11. Find the area in acres of a right-angled triangle, the length of the sides being 24 rods, 7 rods, 25 rods.

12. A courtyard 84 feet by 36 feet is to be paved with flag-stones measuring 6 feet by 3 feet. How many stones will be needed? What will be the cost of the work at $1.25 per square yard?

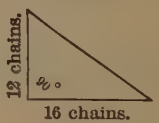
13. Find the length of the fourth side of the following piece of ground. How many yards in the perimeter? How many acres does it contain?



14. Find the perimeter in rods of the field shown in the diagram.

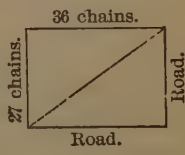
1 chain = 66 feet.

Note. — A right angle contains 90 degrees.



15. What is the length of the diagonal of a rectangular field 90 yards wide, 120 yards long?

16. The dotted line in the accompanying diagram indicates a path through the field. How many yards are saved by taking the path instead of following the road?



17. Find the length (in rods and a decimal) of the diagonal of a square 40-acre field.

CUBE ROOT.

432. To cube a number is to employ it three times as a factor.

The cube of 4, written 4^3, is $4 \times 4 \times 4$, or 64.

Find the cube of 1, 9, 6, 3, 5, 8, 2, 7.

To find the cube root of a number is to find one of the three equal factors of the number.

The cube root of 343, written $\sqrt[3]{343}$, is 7.

The cube of 25, $20 + 5$, is equal to the following:

We have seen (Art. 413) that

$$(20+5)^2 = 20^2 + 2 \times 20 \times 5 + 5^2$$

Multiplying by $20 + 5$ we have

$$\begin{aligned} \text{Product by } 20 &= 20^3 + 2 \times 20^2 \times 5 + 20 \times 5^2 \\ \text{Product by } 5 &= 20^2 \times 5 + 2 \times 20 \times 5^2 + 5^3 \\ (20+5)^3 &= 20^3 + 3 \times 20^2 \times 5 + 3 \times 20 \times 5^2 + 5^3 \end{aligned}$$

which may be written in this way,

$$20^3 + [(3 \times 20^2) + (3 \times 20 \times 5) + 5^2] \times 5.$$

433. Extract the cube root of 15,625.

We see by inspection that the cube root is between 20 and 30; that is, 20 + a second number. Subtract from 15,625 the cube of 20, 8,000. The remainder, 7,625, is equal to the second number multiplied by the sum of three times the square of the first (1,200), etc. Using 1,200 as a trial divisor, the second number is seen to be 6 or less.

$$\begin{array}{rrrr} & & 20+5 & \\ & & \overline{15{,}625} & \\ (20)^3 = & & 8{,}000 & \\ 3 \times 20^2 = & 1{,}200 & 7{,}625 & \text{remainder} \\ 3 \times 20 \times 5 = & 300 & & \\ 5^2 = & 25 & & \\ & \overline{1{,}525} & \underline{7{,}625} & \end{array}$$

Taking 5 as the second number, we add to the 1,200 three times the product of the first and second (300), and the square of the second (25), making a total of 1,525. Multiplying this sum by the second number, we get 7,625, which is equal to the difference between 15,625 and 8,000. The second number is, therefore, 5, and the cube root of 15,625 is 25.

$\sqrt[3]{110,592}$

		$40+8$
		$110,592$
40^3	$=$	$64,000$
3×40^2	$=4,800$	$46,592$
$3\times 40\times 8$	$=960$	
8^2	$=64$	
	$5,824$	$46,592$

Ans. 48.

$\sqrt[3]{658,503}$

		$8\ \ 7$
		$658'503$
8^3	$=$	512
3×80^2	$=19,200$	$146,503$
$3\times 80\times 7$	$=1,680$	
7^2	$=49$	
	$20,929$	$146,503$

Ans. 87.

In the last example we point off three places, beginning at the right, and find the greatest cube in the first period, placing its cube root as the first figure of the answer.

434. Find the cube root of the following:

1. 2,197	6. 238,328	11. $\frac{8000}{12167}$
2. 9,261	7. 421,875	12. 3.375
3. 32,768	8. 551,368	13. $1\frac{217}{512}$
4. 68,921	9. $\frac{512}{729}$	14. $1\frac{397}{1331}$
5. 148,877	10. $\frac{1331}{2744}$	15. $5\frac{451}{729}$

435. Find the cube root of 9,938,375.

When the root contains more than two figures, continue, as shown in the accompanying example, taking for divisor three times the square of the first two figures considered as tens, plus three times the product of the first two figures considered as tens by the third figure, plus the square of the third figure.

		$2\ \ 1\ \ 5$
		$9'938'375$
		8
$3\times 20^2=$	$1\,200$	$1\,938$
$3\times 20\times 1=$	60	
$1^2=$	1	$1\,261$
$3\times 210^2=$	$132\,300$	$677\,375$
$3\times 210\times 5=$	$3\,150$	
$5^2=$	25	
	$135\,475$	$677\,375$

436. Find the value of the following:

1. $\sqrt[3]{1,442,897}$	3. $\sqrt[3]{3,723,875}$	5. $\sqrt[3]{12.977875}$
2. $\sqrt[3]{1,906,624}$	4. $\sqrt[3]{39,651,821}$	6. $\sqrt[3]{66.923416}$

MENSURATION.

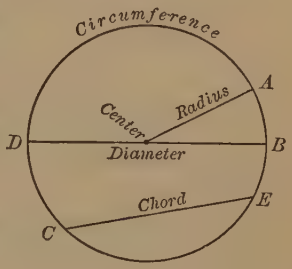
THE CIRCLE.

437. **A *circle* is a plane figure whose boundary is at all points equally distant from the centre.**

The curved line that forms the boundary is called the *circumference*.

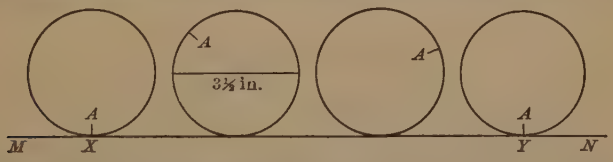
The *diameter* is any straight line drawn from one point of the circumference to another and passing through the centre.

The *radius* is any straight line drawn from the centre to the circumference.



438. Preliminary Exercises.

1. Cut out of stiff paper a circle whose diameter measures $3\frac{1}{2}$ inches. Mark a point A on the circumference, and roll



the circle on a plane surface along the line MN. Make a mark at X where the point A touches the line at the beginning of its revolution, and at Y where A touches the line at the end. Measure the distance XY, which is the length of the circumference of the circle.

2. If the distance between X and Y is 11 inches, what is the ratio of the diameter of a circle to the circumference?

3. Draw several diameters in the circle you have cut out. Measure each. How do the diameters compare?

4. What is the ratio between the diameter of a circle and the radius?

439. Written Exercises.

Find the circumference of a circle whose diameter is $3\frac{1}{2}$ inches.

$3\frac{1}{2}$ inches $\times$ 3.1416 = 10.9956 inches. *Ans.*

The ratio between the circumference of a circle and its diameter has been ascertained to be 3.1416.

PROCESS. — *To find the circumference of a circle multiply the diameter by* 3.1416.

1. Find the circumference of a circle whose diameter is 25 feet.

2. The circumference of a circle measures 39.27 feet. What is the diameter?

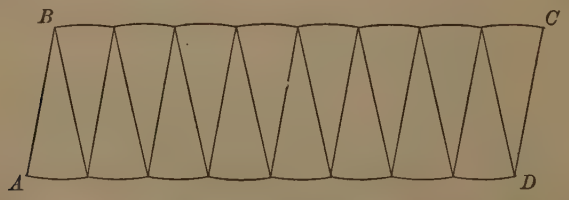
3. The radius of a circle is 8 yards. Find its circumference.

4. The diameter of a bicycle wheel is 28 inches. How far does a bicycle go during 10 revolutions of the wheel?

AREAS OF CIRCLES.

440. Preliminary Exercises.

1. Divide a circle whose diameter measures $3\frac{1}{2}$ inches into sixteen equal parts by cuts passing through the centre in each case. Arrange the parts as shown in the accompanying figure:



2. When the diameter measures $3\frac{1}{2}$ inches, what is the length of AB? What part of the diameter is AB?

3. What part of the circumference is embraced between A and D?

4. When the given circle is divided into a very great number of parts, what will be the length of AD? Of AB?

5. When the number of parts is extremely great, $ABCD$ becomes a rectangle. Find its area.

The number of square inches (feet, etc.) in the area of a circle is obtained by multiplying one-half the number of inches (feet, etc.) in the diameter by one-half the number in the circumference.

This may be expressed by the formula:

$$\text{Area of circle} = \tfrac{1}{2} \text{ diameter} \times \tfrac{1}{2} \text{ circumference.}$$

As the circumference equals diameter $\times$ 3.1416, $\frac{1}{2}$ circumference equals radius $\times$ 3.1416. Multiplying by $\frac{1}{2}$ diameter, or radius, we have:

$$\text{Area of circle} = \text{square of radius} \times 3.1416.$$

To find the area of a circle multiply the square of the radius by 3.1416.

441. Written Exercises.

1. What is the area of a circle whose radius is 36 feet?

$1 \text{ sq. ft.} \times 36^2 \times 3.1416 = 1296 \text{ sq. ft.} \times 3.1416.$

2. Find the area of a circle whose diameter is 50 yards.

3. What is the area of a circle whose circumference is 10 feet?

$$\text{Diameter} = \frac{10}{3.1416}; \quad \text{Radius} = \frac{5}{3.1416}.$$

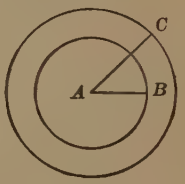
$$R^2 = \frac{5 \times 5}{3.1416 \times 3.1416}; \quad R^2 \times 3.1416 = \frac{5 \times 5 \times 3.1416}{3.1416 \times 3.1416}. \quad \text{Cancel.}$$

4. Calculate the area of a circle whose radius is 1 inch. Of a circle whose radius is 2 inches. What is the ratio of the two areas?

5. What is the ratio between the area of a circle whose radius is 1 inch and that of a circle whose radius is 3 inches?

Indicate operations and cancel.

6. How many square yards are there in a circular walk, the radius, AB, of the inner edge of walk being 10 feet, and that of the outer edge, AC, being 15 feet?

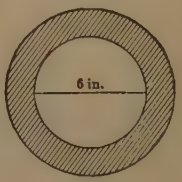


Find the difference between the area of a circle of 15 feet radius, and that of a circle of 10 feet radius.

7. A circular flower bed 20 feet in diameter is surrounded by a walk 5 feet wide. How many square feet of surface does the walk contain?

If you have to subtract 100 times 3.1416 from 225 times 3.1416, how can you shorten the work?

8. How many square inches are there in the surface of a frame 3 inches wide, around a looking-glass 6 inches in diameter?



$$\text{Area} = ? \times 3.1416.$$

9. What is the ratio between the surface of the above frame and that of the looking-glass?

Indicate operations and cancel.

10. What is the radius of a circle whose area is 153.9384 square yards?

11. Find the radius of a circle whose area is 314.16 square inches.

12. Find the area of a circle whose circumference is 15.708 feet.

AREAS OF TRIANGLES.

442. Written Problems.

1. What is the area of a triangle whose sides measure 15, 16, and 17 inches, respectively?

15
16
17
2)48
24 − 15 = 9
24 − 16 = 8
24 − 17 = 7

From the half sum of the three sides subtract each side separately. The square root of the continued product of the half sum and the three remainders will be the area.

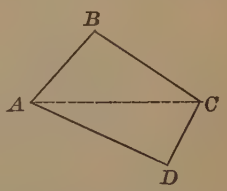
$\sqrt{24 \times 9 \times 8 \times 7} = \sqrt{12{,}096} = 109.98$

Ans. 109.98 square inches.

2. Find the area in square feet of a triangle whose sides measure 35 feet, 84 feet, 91 feet.

3. Find the area of a triangle whose sides measure 21, 28, and 35 rods, respectively.

4. In the following field, AB measures 39 rods; BC, 52 rods; CD, 25 rods; AD, 60 rods; and the diagonal, AC, 65 rods. Find the area of the field in square rods.



Find the area of each triangle separately.

5. Find the area of an isosceles triangle whose base is 30 yards, its equal sides measuring 25 yards.

6. What is the altitude of an isosceles triangle, base, 64 feet, equal sides, 68 feet? Find its area.

7. Find the area of an equilateral triangle, each side being 6 feet.

8. Find the area of a right-angled triangle, base, 42 feet, hypotenuse, 70 feet.

First ascertain the length of the perpendicular.

9. Find the area of an isosceles triangle, altitude, 48 feet, equal sides, 50 feet.

AREAS OF QUADRILATERALS.

443. Written Problems.

1. Find the area of a square whose diagonal is 150 rods.

SUGGESTION. — Calling one side of the square S, we have $S^2 + S^2 = 150^2$, 150 being the hypotenuse of an isosceles right-angled triangle, the other sides being the sides of the square. S^2 is the required area. Do not find the length of S.

2. Find the area of the rhomboid (Fig. 1).

NOTE. — The altitude is the perpendicular of a right-angled triangle having a base of 7 rods and a hypotenuse of 25 rods.

3. Of the rectangle (Fig. 2).

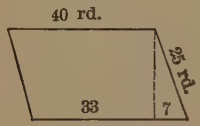


FIG. 1.

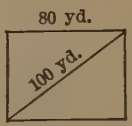


FIG. 2.

4. Of the rhombus (Fig. 3).

5. Of the trapezoid (Fig. 4).

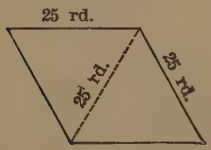


FIG. 3.

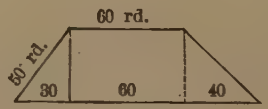


FIG. 4.

6. Of the trapezium (Fig. 5).

7. Of the rhombus (Fig. 6).

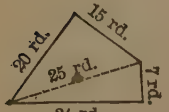


FIG. 5.

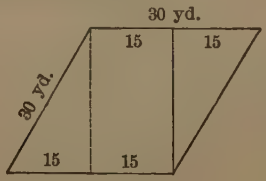


FIG. 6.

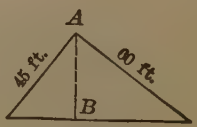


FIG. 7.

8. Find the altitude AB of the preceding triangle (Fig. 7).

(First find the area.)

9. Find the diagonal (in rods) of the square whose area is 5 acres.

10. Find the area of a regular hexagon, composed of six equilateral triangles, each side being 6 inches (Fig. 8).

NOTE. — A plane figure bounded by straight lines is called a polygon. A three-sided polygon is called a triangle. A four-sided polygon is called a quadrilateral. A hexagon is a six-sided polygon. A regular polygon is one having all its sides and all its angles equal. The square is the only regular polygon of four sides.

FIG. 8.

FIG. 9.

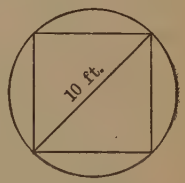


FIG. 10.

11. What is the area of the circle circumscribed about the above hexagon (Fig. 9)?

12. What is the area of the square inscribed in a circle whose diameter is 10 feet (Fig. 10)?

13. Find the area of a square circumscribing a circle whose diameter is 10 feet, and give the ratio of its area to that of the inscribed square.

14. Find the perimeter of a rectangle 80 yards long, the diagonal of which measures 100 yards.

15. A square piece of ground containing 40 acres is divided into 4 square fields of 10 acres each. How many rods of fence will be needed to enclose all the fields?

16. The area of a triangular plot is 480 square yards. Two of the sides are equal in length, and the third measures 32 yards. Find the perimeter.

SURFACES OF PRISMS AND OF CYLINDERS.

NOTE. — The pupils should first examine a number of prisms, right and oblique, regular and irregular, triangular, quadrangular, pentagonal, etc. Right and oblique cylinders should also be at hand.

444. A *prism* **is a body bounded by plane faces, two of which are equal and parallel polygons, the remaining faces being parallelograms.**

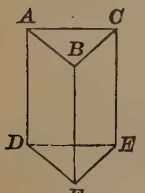


FIG. 1.

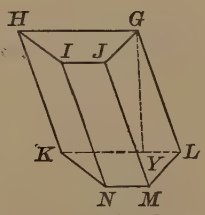


FIG. 2.

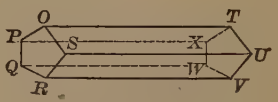


FIG. 3.

The two parallel faces of a prism are called its *bases*. The remaining faces taken together constitute its *convex surface*.

In Fig. 1, *ABC* and *DEF* are the bases; in Fig. 2 the bases are *GHIJ* and *KLMN*; in Fig. 3, *OPQRS* and *TUVWX*.

The sides *AB*, *CE*, etc., *GH*, *IN*, etc., *QR*, *OT*, etc., are called *edges*.

445. Prisms may be either *right* or *oblique*. The convex surface of a right prism consists of rectangles.

Fig. 1 is a right prism; Fig. 2 is an oblique prism.

NOTE. — When a prism is spoken of, a right prism is meant unless the word *oblique* is used.

The *altitude* of a prism is the perpendicular distance between the bases.

AD, *BF*, or *CE* is the altitude in Fig. 1. *GY* is the altitude in Fig. 2.

446. The number of sides in each base determines the name as *triangular* (Fig. 1), *quadrangular* (Fig. 2), *pentagonal* (Fig. 3), etc.

A quadrangular prism whose bases are parallelograms is called a *parallelopipedon*. Fig. 4 is an oblique parallelopipedon. Fig. 5 is a right parallelopipedon. Any two opposite faces of a parallelopipedon may be considered the bases.

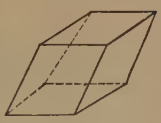


Fig. 4.

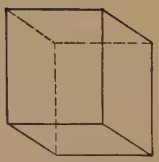


Fig. 5.

447. When the bases are regular polygons, the prism is said to be *regular*.

Fig. 1 is a right regular triangular prism ; Fig. 2 is an oblique irregular quadrangular prism.

448. A cylinder is a body having two circular parallel plane faces, and one curved face.

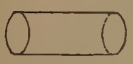


Fig. 6.

The plane faces are the bases. The curved face constitutes the convex surface.

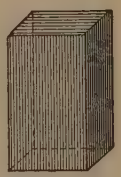
Cylinders, like prisms, are either *right* or *oblique*. The altitude of a cylinder is the perpendicular distance between the bases.

Fig. 7.

Fig. 8.

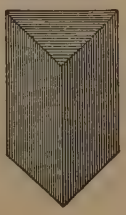
449. Written Problems.

Note. — The pupils should be encouraged to make cardboard models of the forms studied.



1. Find the convex surface of a square prism, one side of its base being 4 inches and its height 6 inches. Draw the development.

Note. — The convex surface is the surface exclusive of the bases.



2. Find the convex surface of a triangular prism, each side of whose base measures 4 inches and whose altitude is 6 inches. Draw the development.

3. Find the convex surface of an hexagonal prism, each side of its base being 4 inches and its altitude 6 inches. Draw the development.

4. Can you show that the convex surface of a prism is found by multiplying the perimeter of the base by the altitude (height)?

5. Find the convex surface of a cylinder, the diameter of its base being 4 inches and its height 6 inches.

To find the convex surface of a right prism (or cylinder) multiply the perimeter (circumference) of the base by the height.

6. How do you find the entire surface of a prism or cylinder?

NOTE. — The entire surface is the surface including the bases.

7. What is the entire surface of a cube whose side is 7 inches? Of a cube whose side is 12 inches?

8. The entire surface of a cube is 216 square inches. What is the length of one side?

SUGGESTION. — Calling the length of one side L, the area of each face will be L^2, and of the six faces, $6\ L^2$. Then, $6\ L^2 = 216$.

9. The convex surface of a cube is 144 square inches. Find the entire surface.

How many faces in the convex surface?

10. Find the entire surface of a square prism, one side of whose base measures 4 inches, and whose altitude is 6 inches.

Entire surface = convex surfaces + areas of bases.

11. The convex surface of a square prism is 600 square feet, the altitude is 15 feet. What is the length of one side of the base?

12. The entire surface of a square prism is 1650 square inches. One side of the base measures 15 inches. What is its convex surface? What is its altitude?

Convex surface = entire surface − area of bases.

SURFACES OF PYRAMIDS AND CONES.

450. A *pyramid* is a body whose convex surface is made up of triangles having a common vertex, the base of the pyramid being a polygon.

FIG. 1.

FIG. 2.

Pyramids are either *right* or *oblique*; *regular* or *irregular*; *triangular*, *quadrangular*, *pentagonal*, etc.

In a right pyramid, each of the triangles that make up the convex surface is isosceles. When, in addition, the pyramid is a regular one, these triangles will be equal to each other.

The altitude of any of these equal triangles constitutes the *slant height* of a right regular pyramid. The *altitude* of the prism is measured by a line drawn from the apex to the centre of the base.

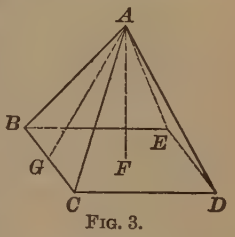


FIG. 3.

AG is the slant height of the square pyramid, Fig. 3. *AF* is its altitude.

451. The *cone* is a body having a single circular base, and a curved convex surface sloping to the apex.

FIG. 4.

In the right cone, Fig. 4, *HI* is the slant height, and *HK* is the altitude. *LO* is the altitude of the oblique cone, Fig. 5.

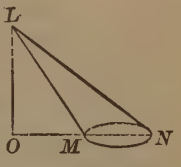
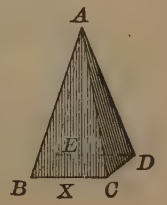


FIG. 5.

452. Written Problems.

1. The convex surface of a square pyramid consists of how many equal triangles? Find the convex surface when one side of its base measures 4 inches and its slant height (*AX*) 6 inches.



2. Draw the development.

To find the convex surface of a pyramid (*or cone*) *multiply the perimeter* (*circumference*) *of the base by one-half the slant height.*

3. Find the entire surface of the above pyramid.

Entire surface = convex surface + area of base.

4. Calculate the entire surface of a square pyramid whose slant height is 18 inches, the area of its base being 144 square inches.

5. Draw the developed convex surface of a cone, the diameter of whose base is 4 inches, and whose slant height is 6 inches.

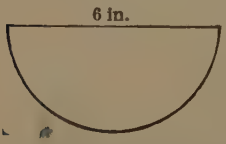
Calculate the convex surface.

6. How many square inches of paper would be required to cover the side and the base of a cone 6 inches in diameter at the base, and having a slant height of 10 inches?

7. Calculate the slant height of a cone whose altitude is 12 inches, the diameter of its base being 10 inches. What is its convex surface?

NOTE. — The slant height is the hypotenuse of a right-angled triangle, the other sides measuring 12 in. and 5 in., respectively.

8. What is the convex surface of a cone, the diameter of whose base is 6 inches, and its slant height 10 inches? Draw the development.



9. A semicircular piece of paper 6 inches in diameter is folded into a hollow cone (without overlapping). What will be the diameter *AB* of the mouth of the cone (the base)? What will be the slant height *BC*?

VOLUMES OF PRISMS AND OF PYRAMIDS; OF CYLINDERS AND OF CONES.

453. Written Problems.

SUGGESTION. — Have the pupils construct of cardboard a hollow square prism of convenient size, and a pyramid having base and altitude respectively equal to those of the prism. Let them use sand or water to ascertain how many times the contents of the pyramid must be taken to exactly fill the prism.

Volume of prism or cylinder = area of base × altitude.

Volume of pyramid or cone = area of base × $\frac{1}{3}$ altitude.

1. Find the volume of a square pyramid, the area of the base being 9 square feet and the altitude 6 feet.

1 cu. ft. × 9 × $\frac{1}{3}$ of 6.

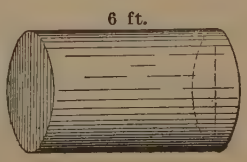
2. What is the volume of a square pyramid whose altitude is 12 inches, one side of the base being 10 inches?

3. The base of a prism is a triangle whose sides measure 3, 4, and 5 inches respectively. Find the solidity, its altitude being 10 inches.

4. The base of a prism 19 feet high is a rectangle whose sides are 9 feet and 13 feet. How many cubic yards does it contain?

5. Find the volume of a prism whose bases are equilateral triangles, each side being 4 feet, and the height of the prism being 12 feet.

6. How many cubic feet are there in a stone roller 6 feet long, 8 feet in circumference?

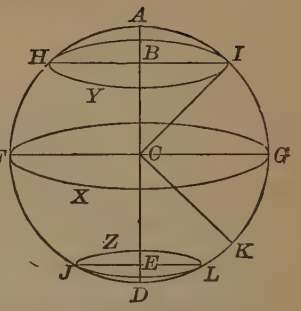


7. Find the volume of a cone whose altitude is 18 meters, diameter of base 6 meters.

SURFACE OF SPHERE.

454. A *sphere* is a body all points on whose surface are equally distant from the centre.

The distance from the centre to the surface is called the *radius* of the sphere. The diameter is a line running between two points on the surface and passing through the centre.

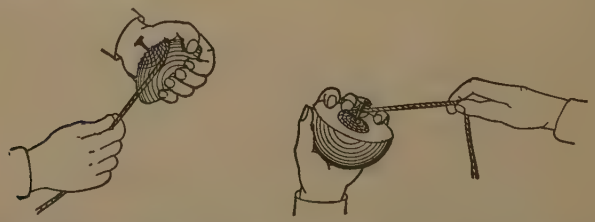


CG, *CK*, *CD*, *CF*, and *CI* are radii; *AD* and *FG* are diameters.

455. If a sphere be cut through at any part, the cut surface will be a circle. When the cutting plane passes through the centre of the sphere, the circle is called a *great circle;* other circles are called *small circles.*

FXGC is a great circle; *HYIB* and *JLEZ* are small circles.

456. Take a wooden hemisphere and drive a tack into the centre of its curved surface. Commencing at the tack, carefully wind a waxed cord about the curved surface, in the way a boy winds a top. When this surface is exactly covered, cut the cord.



Wind the same cord around a tack driven into the plane surface of the base of the hemisphere, pressing it closely to the surface. When the latter is entirely covered, just one-half of the cord will be used.

As a hemisphere is made by passing the cutting plane through the centre of the sphere, its base is a great circle of the sphere.

The above experiment shows that the surface of the hemisphere is equal to that of two great circles of the same sphere.

457. The surface of a sphere is equal to that of four great circles.

Since the surface of a great circle of the sphere is $R^2 \times 3.1416$, the surface of the sphere is $4 R^2 \times 3.1416 = D^2 \times 3.1416$.

To find the surface of a sphere, multiply the square of the diameter by 3.1416.

458. Written Problems.

1. Find the surface of a sphere whose radius is 1 inch.

2. The diameter of a sphere is 2 inches. Find its surface.

3. What is the surface of a sphere whose circumference is 6.2832 inches?

4. At 10 cents a square foot, what will be the cost of gilding a sphere 12 inches in diameter?

5. Find the ratio between the surface of a sphere 1 foot in diameter, and the convex surface of a cylinder 1 foot high, the diameter of the base 1 foot.

6. What is the ratio between the surface of a sphere 1 foot in diameter, and the entire surface of a cylinder 1 foot high, the diameter of the base 1 foot?

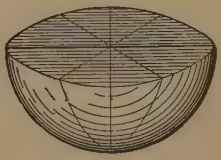
7. Find the surface of a sphere whose circumference is 20 inches.

8. What is the ratio between the surfaces of two spheres whose diameters are 1 inch and 2 inches, respectively?

9. Find the ratio between the surfaces of two spheres whose diameters are 2 feet and 13 feet, respectively.

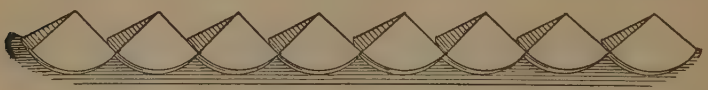
VOLUME OF SPHERE.

459. Cut up a sphere (a round potato, for instance) into a number of small pieces, passing the knife in each case through the centre of the



sphere. Each piece is a solid, having for its base a portion of the surface of the sphere, and for its altitude the radius of the sphere.

When the pieces become very numerous, the base of each may be considered a plane and the solid a pyramid. The volume of each



pyramid is equal to the base $\times \frac{1}{3}$ altitude; and the total volume of all, which is the volume of the sphere, is equal to the total surface of all the bases, which is the surface of the sphere, multiplied by $\frac{1}{3}$ altitude, that is, $\frac{1}{3}$ radius, or $\frac{1}{6}$ diameter.

$$\text{Surface of sphere} = D^2 \times 3.1416,$$

therefore,

$$\text{volume of sphere} = D^2 \times 3.1416 \times \tfrac{1}{6} D = \tfrac{1}{6} D^3 \times 3.1416.$$

To find the volume of a sphere, multiply one-sixth of the cube of the diameter by 3.1416.

460. Written Problems.

1. Find the volume of a sphere whose radius is 3 inches.

1 cu. in $\times$ 36 $\times$ 3.1416.

2. If the diameter of a sphere is 3 inches, what is its volume?

3. What is the ratio between the volumes of two spheres whose diameters are one foot and two feet, respectively?

4. Find the ratio between the volume of a sphere 1 foot in diameter, and that of a cube whose side is 1 foot.

5. The radius of a sphere is 18 inches. What is the circumference of a great circle? The surface? The volume?

6. What is the weight of an iron cannon-ball 12 inches in diameter, considering the weight of a cubic foot of water as 1000 ounces, and considering iron 7.5 times as heavy as water?

7. Find the ratio between the volume of a sphere 4 inches in diameter, and that of a cylinder 4 inches in altitude, diameter of base 4 inches.

NOTE. — *Indicate* the volume of each, and cancel.

8. A man has a cubical block of hard wood, its side measuring one foot, which he wishes made into a sphere one foot in diameter. What decimal part of the block is cut away?

The volume of the sphere is *about* what fraction of the volume of the cube?

MISCELLANEOUS.

461. Written Problems.

1. If a piece of cloth is 20 yards long and $\frac{3}{4}$ yards broad, how broad is another piece of cloth 12 yards long that contains as many square yards as the former?

2. An iron beam 16 feet long, $2\frac{1}{4}$ feet broad, and 8 inches thick, weighs 1280 pounds. What is the length of a similar beam whose breadth is $3\frac{1}{4}$ feet, thickness $7\frac{1}{2}$ inches, and weight 2028 pounds?

3. What will it cost to carpet a room $22\frac{1}{2}$ feet long by $15\frac{3}{4}$ feet wide with carpet $2\frac{1}{4}$ feet wide, costing \$1.50 per yard?

4. What is the length of a box $6\frac{2}{3}$ feet wide and $7\frac{1}{2}$ feet high, that will exactly contain 12 boxes $4\frac{1}{6}$ feet long, $3\frac{1}{3}$ feet wide, and $2\frac{1}{2}$ feet deep?

5. What is the value, at \$120 per acre, of a square field whose side is 35.25 chains?

10 square chains = 1 acre.

6. Find the capacity, in bushels, of a bin 22 feet long, 14 feet wide, 12 feet high?

7. How many gallons will a tank hold, its dimensions being 4 ft. 1 in. by 3 ft. 8 in. by 2 ft. 3 in.?

8. How many square yards are there in the walls and the ceiling of a room 21 feet long, 18 feet wide, 12 feet high?

9. A tank $5\frac{1}{2}$ feet by 6 feet by 7 feet can be emptied by two pipes, one of which discharges 9 gallons per minute and the other 7 gallons per minute. How long will it take each to empty the tank? How long will it take both together?

10. A parlor is 18 feet long, 15 feet wide. Make a diagram, showing how carpet 27 inches wide can be laid without cutting the carpet lengthwise. Which would be the better way to lay carpet 30 inches wide in the above room?

11. Calculate the number of running yards of carpet 30 inches wide needed for the floor of the above room, including $4\frac{1}{2}$ yards wasted in matching the pattern.

Find the cost of carpeting the room at 95 cents per running yard for carpet, 5 cents per square yard for lining, and 10 cents per running yard for sewing and laying.

12. A room is 18 feet wide, 24 feet long, and 9 feet high. There are two doors 4 feet wide, $7\frac{1}{2}$ feet high; two windows 4 feet wide, 6 feet high; and a fireplace 5 feet square. How many square feet of plastering will there be on the walls

and ceiling, deducting for a baseboard 12 inches wide? How many running feet of baseboard will be needed?

Draw "development" of the above room, showing the four walls and the ceiling, and locating the doors, the windows, and the baseboard.

Do not use baseboard where it is not required.

13. At the rate of $1400 for a pile of lumber 25 feet long, 20 feet wide, 10 feet high, what is the value of a pile 50 feet long, 40 feet wide, 20 feet high?

14. If it costs $14 to paint the walls and the ceiling of a room 25 feet long, 20 feet wide, and 10 feet high, what will it cost to paint the walls and the ceiling of a room 50 feet long, 40 feet wide, and 20 feet high?

15. Measure accurately the interior dimensions of a quart or a pint cup, and calculate its volume.

NOTE. — How many cubic inches in a quart, liquid measure?

462. Circular Measure.

60 seconds (″)	1 minute (′).
60 minutes	1 degree (°).
360 degrees	1 circle.

16. If the equatorial circumference of the earth is 25,000 miles, how many miles apart are two places on the equator, the distance between them being 20°?

$$20° = \tfrac{1}{18} \text{ circle.}$$

17. What is the length of a degree on a circle whose diameter is 18 feet?

18. The 60th parallel of latitude is a circle one-half as long as the equator. How many miles due east of Christiania is St. Petersburg, both situated on this parallel, the former being 10° east of Greenwich, and the latter 30° east?

LONGITUDE AND SOLAR TIME.

NOTE. — This topic should be taught in connection with the study of Mathematical Geography. The globe should be used to show the pupils that all places on the same meridian have the same solar time, that a difference in longitude of 15 degrees produces a difference in time of 1 hour, and that the more easterly of two places has the later time.

463. Preliminary Exercises.

1. The difference in time being 1 hour for each 15 degrees, find the difference in longitude between two cities differing in solar time 3 hours.

2. Two places differ in longitude 60 degrees. What is their difference in solar time?

3. London is 75° east of Philadelphia. When it is 1 o'clock at Philadelphia, what is the time at London?

4. When it is 2 P.M. at London, what is the time at Philadelphia?

5. How many degrees of longitude correspond to a difference of 3 hr., 40 min. in solar time?

6. What is the difference in longitude between Philadelphia, 75° west longitude, and St. Petersburg, 30° east longitude?

7. Washington is in 77° west longitude, and uses "standard time," that is, the time of 75° west longitude. What is the difference between the correct time at Washington and its clock time?

8. A town in 84° west longitude uses standard time, that of 90. What is the correct time when the clocks are striking 12, noon?

9. Chicago is 87° 35′ west of Greenwich. Is it earlier or later than noon at Chicago when it is noon at Greenwich? Why?

STANDARD TIME.

464. In 1883, the railroads of the United States adopted a system of dividing the country into four time sections, each of 15° longitude. The 75th meridian west of Greenwich, which passes between New York and Philadelphia, was selected as the starting-point. The section governed by the time of this meridian, called *eastern time*, included the territory between the Atlantic coast and a line drawn through Detroit, Pittsburg, Wheeling, Parkersburg, Hunt-

ington, Bristol, Tenn., Augusta, Ga., and Charleston, these cities being the termini of important railroads. *Central time* is governed by the time of the 90th meridian, and is used by the section west of Detroit, etc., to Bismarck, North Platte, Dodge City, etc. The next section which takes the time of the 105th meridian, called *mountain time*, extends to Helena, Ogden, and the western boundary of Arizona. The rest of the country to the Pacific Ocean takes the time of the 120th meridian, called *Pacific time*.

SOLAR TIME.

465. Written Exercises.

1. Find the difference between the sun time of London and that of Chicago, longitude 87° 35′ west of London.

A difference of 15 degrees of longitude makes a difference of 1 hour; of 15 minutes of longitude, a difference of 1 minute; of 15 seconds of longitude, a difference of 1 second.

If 1 degree of longitude made a time difference of 1 hour, the difference in time between London and Chicago would be 87 hr. 35 min.; as it takes 15 degrees to make a difference of an hour, the difference of time between London and Chicago is $\frac{1}{15}$ of 87 hr. 35 min. Dividing, therefore, 87 hr. 35 min. by 15, we get the time difference as 5 hr. 50 min. 20 sec.

15)87 hr. 35 min.
5 hr. 50 min. 20 sec.

To find the time difference, divide the longitude difference expressed as hours, minutes, and seconds, by 15.

2. When it is midnight at London, what is the sun time at Chicago?

Since the more easterly place has the later time, it is 5 hr. 50 min. 20 sec. before midnight at Chicago. 12 hr. (P.M.) − 5 hr. 50 min. 20 sec. = 6 hr. 9 min. 40 sec. P.M. *Ans.*

3. Two places differ in longitude 37° 18′. What is their difference in solar time?

4. Find the difference in longitude between two places differing in solar time 3 hr. 44 min.

Multiply 3° 44′ by 15.

To find the longitude difference, multiply by 15 the time difference expressed as degrees, minutes, and seconds.

5. Find the difference in sun time between two places in longitude 74° 31′ and 93° 14′ west of Greenwich, respectively.

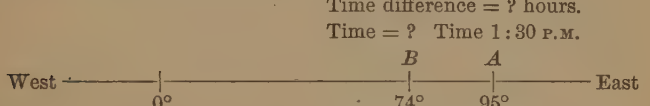
6. When it is noon at a place 11° east of Greenwich, it is 1:30 P.M. at another place. Find the longitude of the latter place.

NOTE. — Owing to the general use of standard time by civilized countries, problems in longitude and time have no practical value except for navigators. The following problems should be worked only after more important topics have been completed.

NOTE. — The word "time" in the following problems means "mean solar time."

7. Given the longitude of A as 95° east, and that of B as 74° east, and the time at A as 1:30 P.M., to find the time at B.

Since the latitude of B has no bearing upon its time, both places may be located upon the same line running east and west.

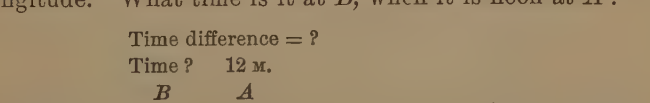


Locate the prime meridian (that of 0°), then the meridians of 74° and 95° east. Mark above the last two the names of the places, B and A. Write above A its given time, 1:30 P.M.

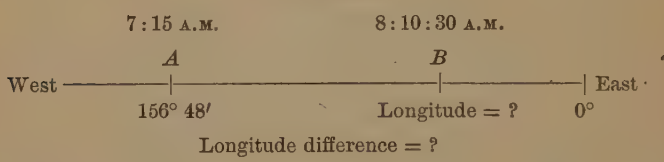
To find the time at B, we must find the difference of time between B and A. The difference in longitude is $95° - 74° = 21°$. The difference in time is 21 hours $\div$ 15.

NOTE. — Remember that the more easterly of the two places has the later time.

8. A is situated in 71° west longitude, B in 107° west longitude. What time is it at B, when it is noon at A?

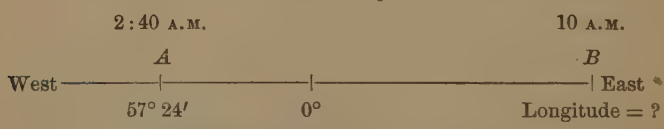


9. Find the longitude of B, whose time is 8:10:30 A.M., when it is 7:15 A.M. at A, whose longitude is 156° 48′ west.



Since B has the later time, its location is east of A. The difference in time, being nearly an hour, shows the difference in longitude to be nearly 15°. Find the exact difference. Is it to be added to 156° 48′ or subtracted from it, to give the longitude of B?

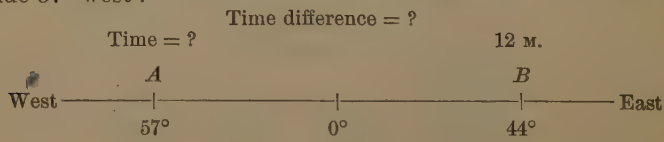
10. When it is 2:40 A.M. at A, in 57° 24′ west longitude, it is 10 A.M. at B. Find the longitude of B.



Longitude difference = 15° × 7⅓ = 110°.

If we go 110° eastward from A, we shall reach the prime meridian after going how many degrees and minutes? How many more degrees and minutes must we travel to reach B? Is B in east or in west longitude?

11. When it is noon at B, what is the time at A, the former being in longitude 44° east, and the latter in longitude 57° west?



Longitude difference = 101°. Why?

Find the longitude or the time:

	Longitude of *A*.	Time at *A*.	Longitude of *B*.	Time at *B*.
12.	63° east	9 A.M.	54° east	?
13.	57° 25′ east	?	83° 20′ east	1:45 P.M.
14.	156° 48′ west	3:15 P.M.	?	4:10 P.M.
15.	?	11:42 A.M.	56° 25′ west	1:27 P.M.
16.	2° 15′ west	6:53 A.M.	67°:48′ east	?
17.	27° 10′ east	?	27° 10′ west	12 M.
18.	?	4:10 P.M.	18° 4′ east	11:30 A.M.
19.	74° 56′ west	3:50 A.M.	?	11 A.M.
20.	4′ 30″ east	8:47 A.M.	90° 15′ west	?
21.	?	10:30 P.M.	32° 30′ east	6:48 P.M.

REVIEW.

466. Oral Problems.

1. At what per cent will \$12, in 3 yr. 4 mo., amount to \$14?

2. What will be the cost of a building lot 100 feet long and 50 feet wide at 50¢ a square foot?

3. A horse was sold for \$90, at which price $12\frac{1}{2}\%$ was gained. What per cent would have been gained by selling him for \$100?

4. What is the premium for insuring \$6000 on my house at $1\frac{1}{4}\%$?

5. How many cubic inches in a ten-inch cube?

6. Bought 2 chairs at \$1.25, one wash-tub for \$1.50, 1 table for \$3.00, and 5 dozen glasses at 48¢ a dozen. Gave a ten-dollar bill in payment. How much change did I receive?

7. My desk is $1\frac{1}{2}$ feet long, and 1 foot wide. How many inches around it?

8. If a man spends 50¢ a day during April, May, and June, what does he spend in the three months?

9. A grocer bought 15 barrels of flour at $5 a barrel. At what price must he sell them to gain $36?

10. Seven-eighths of James's vacation will be equal to seven-ninths of yours; yours will be 63 days. How many will his be?

11. A man sold two cows for $30 each. On one he gained 25%; on the other he lost 25%. Did he gain or lose, and how much?

12. What principal, in three years and 4 months, at 6%, will give $40 interest?

NOTE.—To the following ten problems the wrong answers are very frequently given.

13. Sold a horse for $250, losing $50. What is the loss per cent?

14. If 3 boys solve 3 problems in 3 minutes, how long will it take 6 boys to solve 6 problems?

15. Two boys go fishing; one brings 7 cakes for lunch, the other brings 5 cakes. A third boy joins them at noon, and pays 12¢ for his share of the meal. How should the first two divide the money received?

16. If 100 per cent is gained by selling an article for $1, how much would be gained by selling it for $2?

17. A boy had a slate 5 inches by 7 inches. He buys one twice as large. Give the dimensions of the new slate.

18. A man wishes to put up on the front of his lot a fence 30 feet long. If the posts are 6 feet apart, how much will they cost at 25¢ each?

19. One-half the money taken in by a newsboy is profit. What per cent does he make?

20. 50 per cent of a number multiplied by 30 per cent of the same number equals 60. What is the number?

21. Three-fourths per cent of a number is 90. What is the number?

22. An importer receives some cases of goods numbered consecutively. How many cases are there, if the number of the first is 28, and of the last 75?

467. Written Problems.

1. What is the profit on 9 boxes of oranges, each containing 20 dozen, bought at \$ 1.10 per hundred and sold at the rate of 18 for 25¢?

2. How long will it take a train to go 176 miles at the rate of 3520 feet per minute?

3. If .0375 of an acre of land is worth \$ 9, what is $\frac{3}{32}$ acre worth?

4. At £ 1 1*s.* 7*d.* per barrel, how many barrels of flour can be bought for £ 161 17*s.* 6*d.*?

5. If 580 tiles, each 6 inches square, will cover a certain area, how many tiles, each 4 inches long and 3 inches wide, will be needed to cover the same area?

6. A man receives \$ 1500 commission on his yearly sales. What is the amount of his sales if he is allowed $\frac{1}{4}$ per cent commission?

7. At what rate per cent will \$ 360 produce \$ 3.06 interest in 2 mo. 12 da.?

8. Find the square root of 25.00400016.

9. What will be the capacity, in gallons, of a tank 9 ft. long, 6 ft. 8 in. wide, and 6 ft. 5 in. deep?

10. What decimal multiplied by 312.5 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, .09375, and 2.46?

11. A dealer bought a lot of coal \$ 4.95 per ton. What was the total cost if he gained \$ 142.50 by selling it at \$ 5.25 per ton?

12. Find the value of $\frac{2\frac{1}{9} + 4\frac{5}{12}}{1\frac{3}{4} \times 3} - \frac{1}{6}$ of $6\frac{5}{7}$.

13. The front wheel of a wagon measures 13 feet in circumference. What is the distance travelled in miles, rods, yards, etc., when the wheel has made 527 revolutions?

14. Write in words .349, 300.049, $\frac{340}{9000}$, $300\frac{40}{9000}$.

15. If a bar of silver weighing 4 lb. 6 oz. 12 pwt. is worth £13 8*s*. 4*d*., what is the value (in English money) of a similar bar weighing 7 lb. 9 oz. 12 pwt.?

16. A and B form a partnership. A furnishes \$ 5000; B, \$ 10,000. During the year A draws \$ 1500 of the profits and B draws \$ 1000. At the end of the year the entire business is disposed of for \$ 20,000. What amount should each receive?

17. What per cent is gained on an article bought for 20 per cent less than its value and sold for 20 per cent more than its value?

18. A person loans \$ 750 to M and \$ 1200 to N at the same rate. From the latter he receives half-yearly \$ 9 more interest than from the former. What is the annual rate of interest?

19. A 4-months note for \$ 375, drawn March 19, was discounted at a bank June 4. Find the proceeds. Rate, 6%.

20. M can do a piece of work in 4 days, N can do it in 5 days, O in 6 days. How long will it take the three together to do the work?

1 day $\div$ $(\frac{1}{4} + \frac{1}{5} + \frac{1}{6})$. Analyze.

STOCKS.

468. Some undertakings, such as the construction of a railroad, the building of waterworks, and the like, require more money than any individual may care to risk. It then becomes necessary to secure the coöperation of a number of persons.

The people of a certain town desire to build a street railroad, the construction and equipment of which will require $50,000. The projectors organize a company. If it is desired to interest people of small means, the required *capital* may be divided into *shares* of $10 each, making the total number of shares 5000. If the shares are fixed at $100 each, there will be 500 shares.

To every purchaser of shares, a *certificate* is issued, countersigned by the officers of the company, setting forth the amount of capital, the total number of shares, and the number issued to the holder of the certificate.

At certain fixed periods, quarterly, semi-annually, or annually, the directors of the company determine what part of the profits shall be distributed to the stockholders, the remainder being reserved for new cars, extension of the road, etc. The profits thus distributed are called *dividends*.

469. Written Problems.

1. A company is organized with a capital of $50,000, divided into shares of $100 each. What part of the stock is held by the owner of 10 shares?

2. If dividends of $2000 are distributed at the end of six months, how much should the holder of 10 shares receive?

3. The company announces the dividend as a certain per cent of the capital. What per cent dividend is declared in this case? To what per cent per year is it equal?

4. Mr. H. has $4500 in the savings bank, on which he receives 4 per cent interest. He gives this amount for 30 shares of the stock. What price does he pay per share? What per cent of the par value?

5. If the next semi-annual dividend is 4%, how much more income does Mr. H. receive from the stock than he would obtain from the savings bank?

6. What per cent has Mr. H. received for six months on his investment of $4500?

7. If Mr. H. sells the 30 shares at $164.50 per share, how much more does he receive for it than it cost him?

470. Stocks are generally bought and sold by brokers, who charge, as a rule, $\frac{1}{8}\%$ of the par value for buying or for selling. The prices of the stocks as given in the newspapers are generally a percentage of the par value. Thus, the New York quotation of Pennsylvania R.R. on March 4, 1903, is $151\frac{1}{2}$. This means that the shares of the Pennsylvania R.R. sold for $\$50 \times 1.51\frac{1}{2}$, or $75.75, the par value being $50. The Philadelphia papers of the same date, however, quote the stock at $75\frac{3}{4}$, it being the practice in that city to give the price per share.

471. Written Exercises.

1. Find the cost of 240 shares Anaconda Copper Mining Co., par value $25, at $134\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

$$\text{Cost} = \$25 \times 240 \times (1.34\tfrac{1}{2} + .00\tfrac{1}{8}).$$

To find the cost, multiply the face value of the given number of shares by the rate plus the brokerage.

2. How much brokerage is paid by the buyer of 275 shares bank stock, par value $100, brokerage $\frac{1}{8}\%$?

$$\tfrac{1}{8}\% \text{ of } \$100 \times 275.$$

3. Paid $11,445 for 120 shares Cleveland, Cincinnati Chicago, & St. Louis, par value $100, brokerage $\frac{1}{8}\%$. What was the value of the stock per share?

The brokerage on 120 shares, par value $100, is $\frac{1}{8}\%$ of $12,000, or $15. The cost of the stock is, therefore, $11,430. Dividing by the number of shares gives the value per share.

$$(\$11{,}445 - \tfrac{1}{8}\% \text{ of } [\$100 \times 120]) \div 120.$$

4. Bought 150 shares Evansville and Terre Haute at $69\frac{3}{8}$, brokerage $\frac{1}{8}\%$, paying for it $5212.50. What is the par value per share?

The cost of each share is $5212.50 ÷ 150. Divide this cost by the rate, including the brokerage, $.69\frac{3}{8} + .00\frac{1}{8}$.

$$(\$5212.50 \div 150) \div (.69\tfrac{3}{8} + .00\tfrac{1}{8}).$$

5. A broker sells for a customer 200 shares stock, par value $25, at $102\frac{1}{4}$. If he retains $\frac{1}{8}\%$ brokerage, how much does he pay over to the former owner of the stock?

6. A man buys 60 shares bank stock, par value $100, at 450, no brokerage. If the annual dividend is 18%, what is his income therefrom? What per cent does he receive on his investment?

NOTE. — Dividends are based upon the par value.

7. A manufacturing corporation makes $20,000 a year over all expenses. The stock consists of 4000 shares, par value $50. What rate of dividend can be declared?

What per cent on his investment does a man receive who has bought his stock at 175, no brokerage?

8. A capitalist bought 360 shares stock, par value $25, at $168\frac{1}{2}$. He paid therefor, including brokerage, $15,176.25. What was the rate of brokerage?

9. A broker sold 250 shares, par value $100, at $107\frac{3}{4}$. He deducted brokerage and paid over the proceeds, amounting to $26,875. Find the amount of the brokerage and the rate per cent.

10. A woman invests $35,050 in stock at 175, brokerage $\frac{1}{4}\%$. If the annual dividends are $7\frac{1}{2}\%$, what is her income from the investment?

11. Which investment will pay better, one in a gas company paying 6% dividends annually, their stock selling at 150, the other in a bank paying 7% dividends annually, stock selling at 175?

12. What annual dividend should be declared on railroad stock bought at 125, so that the buyer will receive 4% per annum on his investment? What semi-annual dividend?

13. What will be the cost of 17 shares of canal stock, par value $50, at $93\frac{7}{8}$, and 143 shares gas stock, par value $10, at $102\frac{3}{4}$?

NOTE. — An examination of the prices of stocks as given in the newspapers will show that the rate of dividends constitutes but one consideration influencing buyers. The following prices were offered March 4, 1903, for stocks of four banks, respectively, each of which paid 6 per cent dividends annually; 185, 245, 390, 685. Purchasers of shares of the last three banks evidently hoped for larger dividends in the immediate future.

The values of bonds depend in the first instance upon the character of the corporation issuing them, then upon the rate of interest and the length of time before redemption. United States bonds bring the highest prices, as buyers have no fear of the failure of the government to keep its promises. The following are the prices obtained at the last sales reported to March, 1903:

Rate.	Date of Redemption.	Price Paid.	Last Sale.
U. S. 2's	1930	$108\frac{3}{4}$	Nov. 14, 1902
U. S. 4's	1907	$110\frac{1}{8}$	Feb. 4, 1903
U. S. 4's	1925	136	Feb. 26, 1903
U. S. 5's	1904	103	Feb. 23, 1903

The 5 per cent bonds, although bearing the highest rate of interest, bring only 103, as they will be redeemed at par a little more than a year after they are bought. The purchaser, who paid $103 for a bond, will receive for it in 1904 only $100, with about $5 interest, his net profit for the year being $2 on an investment of $103.

BONDS.

472. A *bond* **is a form of interest-bearing note issued by a corporation.**

A *coupon* bond is one containing certificates of interest which are cut off and presented for payment as interest becomes due. A 10 years' U. S. coupon bond has 40 coupons, one for each quarter-year's interest. Upon each is engraved the date when due, and the sum payable, which is $10 in the case of a $1000 four-per-cent bond.

A *registered* bond contains no coupons, a check for the interest being mailed to the owner, whose name is registered on the books of the corporation.

473. Written Exercises.

1. A railroad company needing more money to extend its road, issues bonds bearing interest at 4%. If these bonds are sold at 95, what rate of interest on the money invested does the owner of a bond receive?

For each $95 invested the owner receives $4 interest. The rate is $4 \div .95$.

To find the rate on the investment, divide the rate of interest by the rate paid for the bond, including brokerage, if any.

2. Find the cost of 20 one thousand dollar bonds at $120\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

3. If the foregoing bonds bear interest at the rate of 6%, what is the annual income? What rate per cent annually is received on the sum invested?

4. A man desires to secure an annual income of $650 for his daughter. What is the face value of 5% bonds necessary to produce this income? What will be the cost of 5% bonds of Denver & Rio Grande at 107, brokerage $\frac{1}{8}\%$?

5. A person desirous of obtaining a semi-annual income of $900 is offered Central Pacific 4's at $99\frac{1}{4}$, Chicago & Alton 3's at $83\frac{1}{4}$, or Western Union $4\frac{1}{2}$'s at $104\frac{3}{8}$, no brokerage in any case. Find the difference between the smallest and the largest outlay necessary to secure the desired income from these bonds.

NOTE. — 4's means bonds paying 4 per cent interest per year.

6. How much money must be invested in the U. S. 2's to yield a quarterly income of $225, bonds selling at $108\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

7. An owner of 6 per cent bonds sells them at the market quotation of 118, and invests the proceeds in $4\frac{1}{2}$ per cent bonds. The latter investment yields him the same income as the former. What did he pay per hundred for the $4\frac{1}{2}$ per cent bonds, no brokerage?

8. A, having a farm of 109 acres, which rents for $681.25 above taxes, etc., sells the same for $200 per acre, and invests the proceeds in U. S. 2's @ $108\frac{7}{8}\%$, brokerage $\frac{1}{8}\%$. Will his yearly income be increased or diminished, and how much?

9. What is the difference in the rate of income obtained from an investment in U. S. 2's at $109\frac{7}{8}$, and one in U. S. 4's at $137\frac{3}{8}$, brokerage $\frac{1}{8}\%$ in each case?

NOTE. — In calculating the rate of interest in the foregoing examples, the time of the redemption of the bonds is omitted from consideration. In the following example, however, the term of the bond is made an element in the computation. The holder of it has received $30 in interest, and he is paid $100 for the bond. Ignoring the matter of compound interest, the question becomes: At what rate will $104 amount in 6 years to $130?

10. Mr. Tower pays $104 for a $100 five per cent bond. At the end of six years the bond is redeemed at par. What rate of interest does he receive on his investment of $104?

DOMESTIC EXCHANGE.

474. Arthur S. Somers, of Memphis, Tenn., wishes to pay John R. Thompson, of The City of New York, $ 3475.86. If Mr. Somers sends a check, drawn on his Memphis bank, Mr. Thompson will be charged a certain sum by his New York bank for collecting the amount of the check, and he will thus receive somewhat less than the sum due him. Mr. Somers, therefore, buys from J. E. Washington, a Memphis banker, who has funds in a New York bank, the following

SIGHT DRAFT.

$ $3475\frac{86}{100}$. MEMPHIS, TENN., Aug. 9, 1904.

At sight, pay to the order of John R. Thompson Three Thousand Four Hundred Seventy-five $\frac{86}{100}$ Dollars, value received, and charge to the account of

JOSEPH E. WASHINGTON.

To Chemical Bank,
The City of New York.

Mr. Somers is charged for this draft a premium of $ 1.50 per $ 1000; that is, he pays Mr. Washington $ 1001.50 for each $ 1000. The cost of the draft is, therefore, $ 3475.86 × 1.0015, or $ 3481.07.

475. Exchange is at a *premium* when the cost of a sight draft is greater than its face; it is at a *discount* when the cost of a sight draft is less than its face.

476. Mr. Thompson could collect the sum due him by making a draft on Mr. Somers as follows:

TIME DRAFT.

$ $3475\frac{86}{100}$. NEW YORK, Aug. 9, 1904.

At three days' sight pay to the order of The National Bank of Commerce Three Thousand Four Hundred Seventy-five and $\frac{86}{100}$ dollars, value received, and charge to the account of

JOHN R. THOMPSON.

To Arthur S. Somers,
Memphis, Tenn.

Mr. Thompson deposits the draft in the National Bank of Commerce for collection. This bank forwards it to a Memphis bank. The latter notifies Mr. Somers. If he wishes to pay the draft at the expiration of three days, he writes across the face in red ink, "Accepted," with the date, "Aug. 11, 1904," and adds his signature. Aug. 14 he pays the money to the Memphis bank, which notifies the Bank of Commerce, and the sum is placed to the credit of Mr. Thompson, less the cost of collection.

477. A *sight* draft is payable upon presentation, except in those states allowing "days of grace." A *time* draft is one payable a specified number of days after acceptance. In some states three additional "days of grace" are allowed.

478. Written Exercises.

1. Find the cost of a Boston draft on New York for \$ 1875, at 12 ¢ discount per \$ 1000.

Face	\$ 1875.
Discount \$ 1875 × .00012	.225
	\$ 1874.775

Ans. \$ 1874.78.

To find the cost of a sight draft, add the premium to the face, or subtract the discount from the face.

2. What will a St. Louis merchant pay for a draft on New York for \$ 2460.53, at 50 ¢ premium per \$ 1000?

3. At $\frac{1}{8}\%$ premium, find the cost of a sight draft for \$ 1843.60.

4. At 75 ¢ discount per \$ 1000, how much will cost a sight draft on Milwaukee for \$ 946.75?

5. Paid \$ 632.18 for a sight draft on Milwaukee. What was the face of the draft, the discount being $\frac{3}{40}\%$?

BILLS OF EXCHANGE.

479. Bills of exchange are either domestic or foreign. A domestic bill of exchange is called a *draft*, the term *bill of exchange* being generally applied only to foreign bills.

480. Fred Johnston owes John Ahern & Co., of London, £180 17*s.* 6*d.* He buys from John Cottier & Brother a bill of exchange drawn on their London correspondent. The bill is drawn in duplicate, one being sent by Mr. Johnston to John Ahern & Co., and the other being retained by the former to send in case of the loss of the first. When either is paid the other becomes of no value.

The following is the form of the first of a set of exchange.

Exchange for £180 17*s.* 6*d.* NEW YORK, Dec. 14, 1903.

Sixty days after sight of this First of Exchange (Second unpaid), pay to the order of John Ahern & Co., One Hundred Eighty Pounds Sterling, Seventeen Shillings Six Pence, value received, and charge the same to account of

To James Lennon & Co., London. No. 39.

JOHN COTTIER & BROTHER.

Upon receipt of this bill, John Ahern & Co. present it for acceptance. They receive the money sixty days thereafter.

481. Written Exercises.

1. Find the cost of the above bill at $4.87 per pound.

£200 = $974.00
20 = 97.40
£180 = $
10*s.* = 2.435 £½
5*s.* =
2*s.* 6*d.* =
$

2. What is the cost of a cable transfer of £ 251 11*s.* 9*d.*, at $ 4.88½ per pound?

£ 250 = $ 1221.25 ¼ of £ 1000
1 =
10*s.* =
1*s.* =
6*d.* =
3*d.* =

The newspapers give quotations of foreign exchange for sight and 60-day bills, also for cable transfers.

482. The New York quotations for French exchange give the number of francs for $ 1.

Paris cable transfers	5.16¼ @ 5.15⅝.
Paris bankers' 60 days	5.18¾ @ 5.18⅛.
Paris bankers' sight	5.16⅞ @ 5.16¼.

The quotations for German exchange give the value in U.S. money of 4 Reichmarks (or marks).

Reichmarks (4) 60 days	95⅛ @ 95¼.
Reichmarks (4) sight	95⅜ @ 95½.

3. Find the cost of a sight bill on Paris for 1000 francs, at 5.16¼ francs for $ 1.

4. Find the cost of a 60-day bill of exchange on Berlin for 1874.35 marks, at 95¼ ¢ for 4 marks.

5. What will be the face in marks of a sight bill of exchange on Berlin that can be bought for $ 1000, at 95½ ¢ for 4 marks?

6. A New York merchant pays $ 1637.50 for a 60-day bill on Paris. What is the face of the bill, the rate of exchange being 5.18½ francs for $ 1?

7. At $4.88 per pound, what will be the face of the sight bill on London that can be bought for $1500?

$$\frac{\cancel{1500.00}}{\cancel{4.88}\ 61} = \frac{18750}{61}$$

£307 7*s.*, etc.
61)18750
450
£23 remainder
20
460*s.*, new dividend

8. Bought goods in London amounting to £437 5*s.* 10*d.* less 4%. How much do I pay in Boston for a sight bill of exchange at $4.88½ to settle the account?

9. What will be the cost in Chicago for a 60-day bill on Paris that will pay for the following articles? Rate, 1 franc = 19½¢.

18 pieces silk, 44 meters each, at 25 francs per meter, less 7½%.

3 pieces of cloth, 50 meters each, at 20 francs per meter, less 5%.

Packing charges, 60.50 francs.

10. I wish to send a sight bill of exchange on Berlin in payment of the following invoice:

4 cases musical instruments, amounting to 3598.60 marks, less 10, 5, and 2½%.

Freight to Hamburg, 165 kilos, at 4.80 marks per kilo.

At 95⅞¢ for 4 marks, what will be the cost of the bill of exchange?

11. If the rate of exchange is 50¢ discount per $1000, what is the face of the sight draft on Boston, that can be bought in New York for $1000?

NOTE. — $999.50 in New York will buy a sight draft on Boston for $1000.

12. When the premium is $1.25 per $1000, Mr. Brown pays $1634.04 for a draft on Louisville. What is the face of the draft?

COMPOUND INTEREST.

483. Compound Interest is interest on the principal and on the unpaid interest, which is added to the principal at regular intervals. The interest may be compounded annually, semi-annually, or quarterly, according to agreement.

Compound interest is allowed by savings banks. It is not collectible on notes, mortgages, or the like.

484. Written Exercises.

1. Find the amount of $375, for 1 year, at 6%. Considering this as a new principal, find the amount for a year, same rate. Find the amount of this last principal for 3 months.

2. What is the amount of $375, for 2 yr. 3 mo., at 6%, compound interest?

3. What is the amount of $375, for 2 yr. 3 mo., at 6%, the interest compounded semi-annually?

Principal, $375.

3%	11.25	6 months' interest.
	386.25	Amount 6 months.
3%	11.5875	6 months' interest.
		Amount 1 year,
3%	_____	6 months' interest.
		Amount 1½ years.

etc., etc., etc.

4. Find the compound interest on $375, for 2 yr. 3 mo., at 6 per cent, compounded semi-annually.

NOTE. — To find the *compound interest*, deduct $375 from the *amount* for 2 yr. 3 mo.

5. What is the amount of $100, at compound interest, for 3 years, interest at 6%, compounded annually?

ANNUAL INTEREST.

When the maker of a note fails to keep his contract to pay interest annually, the laws of some states, including Michigan, permit the collection of *simple interest* on the deferred payments of interest.

485. Written Problems.

1. Find the amount due June 1, 1908, on the following note, no payments of principal or interest having been made:

DETROIT, MICH., June 1, 1904.

Four years after date, without days of grace, I promise to pay to the order of Daniel W. Lawler, Six Hundred Dollars, value received, with annual interest at six per cent.

$\$ 600\frac{00}{100}$. GEORGE OXNARD.

Principal,	$600.00
Interest, 4 years, at 6%,	144.00
3 years' interest, at 6%, on the 1st year's interest, $36,	6.48
2 years' interest, at 6%, on the 2d year's interest, $36,	
1 year's interest, at 6%, on the 3d year's interest, $36,	
Amount due June 1, 1910,	$

Find the interest on the principal for the entire time, and on each annual interest for the time it remained unpaid. The sum of the principal and all the interest is the amount due.

2. Find the amount due, at 5%, for 5 years, on a note for $1200, annual interest being unpaid.

3. The maker of a note for $900, with annual interest at 7%, makes the first and the second interest payments when due. How much will he owe at settlement, 6 years after the date of the note?

4. Find the difference between the amount due at 6% for 3 years on a note for $300, annual interest unpaid, and the amount of the same sum placed at compound interest for the same time at the same rate.

5. What is the amount of a note for $720, at 4 years, at $4\frac{1}{2}\%$, annual interest unpaid after the first year?

6. Find the amount due March 1, 1906, on a note for $500, dated March 1, 1900, with interest at 6%, annual interest unpaid after the third year.

METRIC SYSTEM.

486. The metric system, which is used in nearly all the countries of continental Europe, is based upon the meter. The length of the meter is one ten-millionth part of the length of the meridian from the equator to the poles — about 39.37 inches.

The subdivisions of the meter are denoted by the Latin prefixes milli ($\frac{1}{1000}$), centi ($\frac{1}{100}$), deci ($\frac{1}{10}$). For the multiples, the Greek prefixes deka (10), hecto (100), kilo (1000), and myria (10,000) are used.

487. It will be noticed, in the table below, that small letters are used for the abbreviations of the Latin prefixes of the subdivisions, and capital letters for the Greek prefixes of the multiples. The following is the table of

488. Measures of Length.

10 millimeters (mm)	1 centimeter (cm)
10 centimeters	1 decimeter (dm)
10 decimeters	1 *meter* (m)
10 *meters*	1 dekameter (Dm)
10 dekameters	1 hectometer (Hm)
10 hectometers	1 kilometer (Km)
10 kilometers	1 myriameter (Mm)

The units of this table in common use are the centimeter, the meter, and the kilometer.

Long distances are expressed in kilometers. The thickness of wire is given in millimeters.

489. Written Problems.

1. What will be the cost in francs of 380 m 75 of dress goods at 2 f 60 per meter?

380 m 75 is read 380 meters 75 centimeters. It is also written 380.75 m, but the first method is the more common one in Europe. 2 f 60 is read 2 francs 60 centimes. A period (.) is not used after the abbreviations of meter, liter, franc, etc.

2. How many square meters in a piece of carpet 26 m 50 long, 85 cm wide?

3. How many square meters in a circle whose diameter is 15 meters?

4. An *are* is a surface 10 meters long, 10 meters wide. How many ares in a field 135 meters long, 69 meters wide?

5. Find the area in ares of a right-angled triangle whose base is 245 meters, hypotenuse 875 meters.

6. A *stere* is a cubic meter. What will be the cost, at 8 f 50 per stere, of a pile of wood 10 meters long, 1 meter wide, 3.25 meters high?

7. A cube one decimeter each way contains a liter (l), which is the principal unit of dry and liquid measure.

How many liters' capacity has a tank 10 m 50 long, 8 m wide, 6 m 50 high?

Change each dimension to decimeters.

8. How many bottles, each containing 0 l 75, can be filled from a hogshead containing 222 l?

9. How much will be received for 36 bags of beans, each containing 68 liters, at 1 mark 25 per dekaliter?

10. A liter of water weighs a kilogram (1000 grams). How many kilos of oil would a tank contain, its dimensions being 5 meters by 4 meters by 3 meters, the weight of the oil being 92% of the weight of water?

11. Assuming the length of the meter as 39.37 inches, what is the length of the kilometer in yards?

490. Measures of Surface.

100 sq. mm = 1 sq. cm
100 sq. cm = 1 sq. dm
100 sq. dm = 1 sq. m = 1.196 sq. yd.

491. The square meter is the principal unit of surfaces, such as walls, ceilings, floors, etc.

100 centiares (ca) = 1 are (a) = 119.6 sq. yd.
100 ares = 1 hectare (Ha) = 2.47 acres.

The are is the principal unit of surface of small plots of land. The area of a farm is expressed in hectares, of a country in square kilometers.

492. Measures of Volume.

1000 cu. mm = 1 cu. cm
1000 cu. cm = 1 cu. dm
1000 cu. dm = 1 cu. m = 35.316 cu. ft.

The principal unit is the cubic meter.

493. The *stere* (cubic meter) is used for measuring wood.

10 decisteres (dst) = 1 *stere* (st) = 35.316 cu. ft.
10 *steres* = 1 dekastere (Dst).

The stere is the only unit used.

494. Dry and Liquid Measures.

10 milliliters = 1 centiliter.			
10 centiliters = 1 deciliter.		Dry.	Liquid.
10 deciliters = 1 *liter* (l)	=	.908 qt.	= 1.057 qt.
10 *liters* = 1 dekaliter	=	1.135 pk.	= 2.642 gal.
10 dekaliters = 1 hectoliter	=	2.837 bu.	= 26.417 gal.
10 hectoliters = 1 kiloliter.			
10 kiloliters = 1 myrialiter.			

The liter and the hectoliter are the principal units.

495. Table of Weight.

10 milligrams (mg)	= 1 centigram.
10 centigrams	= 1 decigram.
10 decigrams	= 1 *gram* (gr).
10 *grams*	= 1 dekagram.
10 dekagrams	= 1 hectogram.
10 hectograms	= 1 kilogram (kilo) = 2.2046 lb.
10 kilograms (Kg.)	= 1 myriagram.
10 myriagrams	= 1 quintal.
10 quintals	= 1 tonneau (ton).

The kilo is the ordinary unit. Heavy articles are sold by the tonneau.

496. Written Exercises.

1. The Eiffel tower is 300 meters high. What is its height in feet?

2. The Danube is 2600 kilometers long. Find its length in miles.

3. A bottle filled with water weighs 1.170 kilos; the weight of the bottle is 420 grams. What is the capacity of the bottle in liters?

4. Find the weight in kilos of 15 liters of olive oil, which weighs .915 time as much as water.

5. A rectangular field 123 meters long, and 85.5 meters wide, yielded 13.25 hectoliters of wheat per hectare. The wheat weighed 84.350 kilos per hectoliter and sold for 23.50 francs per 100 kilos. What sum did the crop bring?

6. What will be the cost in francs of papering a room 5 m 42 long, 4 m 18 wide, and 3 m 10 high, at 1 f 20 per square meter?

7. Calculate the expense of building a wall 14 m 50 long, 7 m 80 high, 0 m 22 thick, of bricks 0 m 22 long, 0 m 11 wide, 0 m 06 thick, the bricks costing 58 francs per thousand and the labor, etc., 32 f 80 per cubic meter.

8. Find the profit on a pile of wood 20 meters long, 4 meters wide, 8 meters high, bought at 12 francs per stere, and sold at 4 francs per 100 kilos, the weight of the wood being .42 times the weight of water.

9. A liter of wheat weighs 760 grams. When ground it produces 89 per cent flour and 11 per cent bran. Find the weight of the flour that can be made from the wheat contained in a bin 2 m 60 long, 2 m 40 wide, and 1 m 50 deep. Find the value of the wheat at 4 f 85 per double dekaliter.

10. If sea water contains $\frac{1}{40}$ of its weight of salt, how many hectoliters of sea water should be evaporated to obtain 100 kilos of salt, a liter of sea water weighing 1.026 kilos?

REVIEW.

497. Oral Problems.

1. If $\frac{2}{3}$ yard costs \$4.50, what will $\frac{4}{5}$ yard cost?

2. If 3 men can do a piece of work in 4 days, how long will it take 24 men to do it?

3. What principal at interest for 5 years, at 6 per cent, will produce \$12, simple interest?

4. A stack of hay will keep a cow 20 weeks, or a horse 15 weeks. How long will it keep them both?

NOTE. — What part will each eat in a week? What part will both eat in a week?

5. How many days from May 16 to July 5?

6. Sold a cow for \$24, losing thereby 40% of the cost price. Had I sold her for $33\frac{1}{3}$% advance on the cost, what should I have received for her?

7. What will 460 pounds of tea cost at \$.48 per pound?

8. If 12 ounces of bread are destroyed in making a gill of whiskey, how much will be destroyed in making a gallon?

4 gills = 1 pint.

9. If the weight of air is 15 pounds on the square inch, what is it on the square foot?

10. Seven is three-fifths of what number?

11. What is the value of 960 pounds of wheat at $1.05 per bushel of 60 pounds?

12. At what rate per cent will $400 make $37.50, simple interest, in 1 yr. 3 mo.?

13. What is the brokerage on $10,400, at $1\frac{3}{8}\%$?

14. What will 3280 feet of lumber cost @ $25 per thousand?

15. A and B are partners; A puts in $\frac{5}{12}$ of the stock, and B the remainder; B's gain is $1400. Find A's gain.

16. What is the difference in the longitude of two places whose difference in sun time is two hours and three minutes?

17. A room is $\frac{3}{5}$ as wide as it is long. Its length is 20 feet. How many square feet are there in the floor?

18. If 5 yards of cloth cost 90¢, what will $\frac{5}{9}$ of a yard cost?

19. An agent insured a house for me at a commission of $\frac{1}{4}\%$. His commission was $15. For how much was the house insured?

20. A gold-digger who had 3 pounds of gold dust, lost 9 ounces. What per cent was left?

498. Written Problems.

1. What number must be added to the sum of $\frac{4}{5}$, $\frac{7}{8}$, and $\frac{11}{12}$ to make $5\frac{89}{120}$?

2. Find the interest on $2320, for 5 months and 21 days, at the rate of 7 per cent a year.

3. Find the interest on $640, from Sept. 3, 1904, to Oct. 30, 1905, at 6 per cent per annum.

4. At compound interest, what will $200 amount to in 1 year and 3 months, at 6 per cent, interest compounded semi-annually?

5. A man drew out of the bank $\frac{3}{5}$ of his money, and expended 30% of 50% of this for 936 bushels of wheat at $0.87\frac{1}{2}$ a bushel. What sum had he left in bank?

6. A house that cost $14,500 rents for $1189. What per cent does it pay on the investment?

7. If 4 men dig a ditch 24 rods long in 20 days, how long a ditch can 5 men dig in 8 days?

8. For what sum must a 60-day note be written to yield $294.75 at a bank, discounting at 6%?

9. An agent receives $5616 for silk he has purchased and his commission on it at 4%. How many yards did he purchase at $1.50 per yard?

10. What will be the proceeds of a 60-day note for $500, dated June 4, 1904, and discounted at a bank July 1, 1904, at 6%?

11. At what rate will $142 gain $21.30 interest in 3 years?

12. What is the duty, at 50¢ a pound and 30% ad valorem, on 700 yards of French broadcloth, invoiced at $1.25 per yard, and weighing $1\frac{1}{4}$ pounds per yard?

13. What will be the amount, at compound interest, of $340, at 8%, for 1 yr. 3 mo., the interest compounded semi-annually?

14. If I lose 10% by selling goods at 18¢ a yard, for what must they be sold to gain 20%?

15. I sold $24\frac{1}{2}$% of my estate, or $1372 worth. I am worth, in addition to my real estate, $14,000. How much am I worth in all?

REVIEW OF FRACTIONS.

499. Oral Exercises.

Give products:

1. $84 \times 24 = 25$ times $84 - 84 = 2100 - 84$.
2. 48×24.
3. 24×36.
4. 48×49.
5. 84×49.
6. 84×74
7. 48×74
8. $84 \times 24\frac{1}{2} = 25$ times $84 - \frac{1}{2}$ of $84 = 2100 - 42$.
9. $48 \times 24\frac{1}{2}$.
10. $36 \times 24\frac{1}{2}$.
11. $48 \times 24\frac{3}{4}$.
12. $36 \times 24\frac{3}{4}$.
13. $48 \times 24\frac{7}{8}$.
14. $36 \times 24\frac{2}{3}$.
15. $48 \times 36\frac{1}{2} = 37\frac{1}{2}$ times $48 - 48 = (\frac{3}{8}$ of $4800) - 48$.
16. $48 \times 11\frac{1}{2}$.
17. $48 \times 86\frac{1}{2}$.
18. $48 \times 37\frac{1}{4}$.

Give quotients:

1. $36 \div \frac{1}{4}$.
2. $36 \div \frac{3}{4}$.
3. $36 \div 2\frac{1}{4}$.
4. $36 \div \frac{1}{3}$.
5. $36 \div 1\frac{1}{3}$.
6. $36 \div 1\frac{1}{2}$.
7. $18\frac{3}{5} \div 3$.
8. $20\frac{1}{6} \div 4$.
9. $17\frac{1}{2} \div 5$.
10. $19\frac{1}{5} \div 6$.
11. $16\frac{1}{3} \div 7$.
12. $17\frac{2}{3} \div 8$.
13. $12\frac{1}{4} \div 1\frac{3}{4}$.
14. $16\frac{1}{2} \div 1\frac{1}{2}$.
15. $13\frac{1}{3} \div 3\frac{1}{3}$.
16. $14\frac{2}{5} \div 1\frac{1}{5}$.
17. $15\frac{3}{4} \div 2\frac{1}{4}$.
18. $17\frac{1}{2} \div 3\frac{1}{2}$.

500. Written Exercises.

Find products:

1. $648 \times \frac{7}{8}$.
2. $976 \times \frac{15}{16}$.
3. $1648 \times 87\frac{1}{2}$.
4. $2592 \times 91\frac{5}{16}$.
5. 2416×875.
6. 874×99.
7. 848×125.
8. 375×999.
9. 792×25.
10. 457×16.
11. 1864×250.
12. 983×51.
13. $1576 \times 62\frac{1}{2}$.
14. $176 \times 23\frac{3}{4}$.
15. 1128×375.
16. $895 \times 44\frac{4}{5}$.

501. Written Exercises.

1. Divide the sum of $6\frac{3}{7}$ and $1\frac{7}{8}$ by the difference between $2\frac{1}{5}$ and $3\frac{1}{3}$.

2. What is the difference between the sum of $\frac{3}{7}$ and $\frac{5}{9}$ and the product of $\frac{4}{5}$ and $\frac{7}{12}$?

3. What is the product of the sum and the difference of $4\frac{1}{4}$ and $6\frac{1}{3}$?

4. Subtract $\frac{2}{3}$ of $\frac{1}{4}$ from $\frac{11}{18}$; and find the value of $\frac{3}{11}$ of 16*s.* 6*d.*

5. Add $7\frac{5}{9}$, $\frac{3}{4}$ of $\frac{7}{12}$, $\frac{5}{8}$ of $7\frac{2}{3}$, and $\frac{19}{24}$.

6. Reduce $\frac{3}{5}$ of a square rod to the fraction of an acre, and find the value of $\frac{7}{64}$ of a ton in pounds and ounces.

7. Reduce $\frac{696}{1305}$ to its lowest terms, and $\dfrac{3\frac{1}{3}-2\frac{1}{5}}{3\frac{1}{3}+2\frac{1}{5}}$ to its simplest form.

8. Add $\frac{1}{2}$, $\frac{5}{9}$, $\frac{3}{4}$, and $\frac{1}{3}$; multiply the sum by $\frac{9}{22}$; and subtract the product from 1.

9. Find the value of $9\frac{4}{11}$ meters at $4\frac{5}{9}$ francs per meter.

10. Divide $2\frac{1}{4}$ by $3\frac{1}{2}$, and add the quotient to $\frac{5}{14}$.

11. Multiply $2\frac{2}{13}$ by $16\frac{5}{7}$, and divide the result by $1\frac{1}{2}$ of $2\frac{2}{5}$.

12. Reduce 7*s.* 6*d.* to the fraction of a pound, and 7 hr. 12 min. to the fraction of a day.

13. Reduce to its simplest form $\dfrac{2+\frac{1}{6}\text{ of }5\frac{1}{7}}{\frac{5}{7}\text{ of }\frac{1}{2}}$.

14. Add together £$\frac{5}{8}$ and $\frac{1}{10}$ of $5\frac{5}{9}$ shillings.

15. What fractional part of 7 A. 127 sq. rd. is 5 A. 81 sq. rd.?

16. What must be added to $\frac{3}{7}$ of $\frac{5}{8}$ to make it equal to $\frac{9}{10}$ of $3\frac{3}{4}$?

17. $\frac{2}{5}$ of a number is 148. What is the number?

18. If $\frac{5}{8}$ of a field is worth \$325, what is the field worth?

19. If $\frac{2}{3}$ of a house is worth \$4900, what is the value of $\frac{1}{8}$?

REVIEW OF DENOMINATE NUMBERS.

502. Written Exercises.

1. Change 43 yards to rods and a fraction.
2. Change 43 yards to rods and yards.

43 yards ÷ $5\frac{1}{2}$ yards gives the *number* of rods.

3. Change 43 yards to rods, yards, and feet.
4. Change 43 yards to rods, yards, feet, and inches.
5. Change 72 yards to rods, etc.
6. Change 66 yards to rods.

Change to rods, yards, etc.:

7. 49 yards.
8. 147 feet.
9. 1764 inches.
10. $8\frac{10}{11}$ rods.
11. 1836 inches.
12. 1837 inches.
13. 52 yards.
14. $49\frac{1}{2}$ yards.

503. Change to rods, etc.:

15. 1483 inches.
16. 984 inches.
17. 1345 inches.
18. 2796 inches.
19. 1121 inches.
20. 1470 inches.
21. 3453 inches.
22. 1278 inches.
23. 1576 inches.

504. Add:

24.
4 rd.	3 yd.	1 ft.
9 rd.	4 yd.	2 ft.
3 rd.		1 ft. 6 in.

25.
5 rd.	4 yd.	2 ft.
	5 yd.	1 ft.
6 rd.	1 yd.	

26. From 8 rd. 1 ft. take 2 rd. 2 ft.
27. Find the difference between 3 rd. 1 yd. 1 ft. and 16 rd.
28. Multiply 5 rd. 4 yd. 2 ft. by 4.
29. Multiply 11 rd. 2 ft. by 10.
30. Divide 30 rd. 5 yd. 2 ft. by 8.
31. Divide 34 rd. 2 yd. by 9.

REVIEW OF COMMERCIAL DISCOUNT.

505. Oral Exercises.

When the list price is $1, what is the net price after the deduction of each of the following discounts?

1. 30 and 20%.

The net price after a deduction of 30% is 70% of $1, or 70¢. Deducting 20% of 70¢ leaves 80% of 70¢, or 56¢.

2. 40 and 10%.

The net price is 60% of 90% of $1, or 54% of $1.

3. 25 and 40%.

4. 50 and 10%.

5. 40 and 20%.

6. 10 and 5%.

7. 20 and 20%.

8. $33\frac{1}{3}$ and 10%.

9. 20 and 15%.

10. 30 and 15%.

506. What single discount is equal to each of the following double discounts?

11. 30 and 30%.

The net price is 70% of 70% of list price, or 49% of list price. The discount is, therefore, 100% − 49%, or 51%.

12. 20 and 25%.

13. 25 and 20%.

14. 15 and 30%.

15. 40 and 30%.

16. 30 and 10%.

17. 40 and 5%.

18. 50 and 20%.

19. 40 and 15%.

20. 50 and 15%.

Find the single discount equal to each of the following:

21. 50 and 20 and 10%.

22. 40 and 25 and 20%.

23. 10 and 10 and 10%.

24. 30 and 20 and 10%.

507. Written Exercises.

Which is the better discount for the buyer?

1. 30 and 20%, or 40 and 10%.

30 and 20% off = 70% of 80% net, or 56% net. 40 and 10% off = 60% of 90% net, or 54% net. The latter is the better for the buyer.

2. 50 and 10%, or 40 and 20%.
3. 20 and 20%, or 30 and 10%.
4. 20 and 15%, or 30 and 5%.
5. 30 and 15%, or 25 and 20%.
6. 30 and 30%, or 50 and 10%.
7. 40 and 30%, or 20 and 50%.
8. 40 and 5%, or 30 and 15%.
9. 20 and 50%, or 60 and 10%.
10. 40 and 15%, or 30 and 25%.

REVIEW OF INTEREST.

508. Six Per Cent Method.

Interest is the product of the *principal* by the *rate* expressed as hundredths by the *time* in years and fraction. The usual method is to perform the operations in the above order. When the rate is 6%, some prefer to first multiply the rate by the time, and to use this as a multiplier of the principal.

In finding the product of the rate by the time, advantage is taken of the fact that 6 is a factor of 12 and 30. Six per cent a year is $\frac{1}{2}$ per cent a month and $\frac{1}{60}$ per cent a day.

When the rate is a different per cent, the interest is first obtained at 6 per cent by this method, and from this result the interest is calculated for the given rate.

509. Find the interest on \$2874.35 at $3\frac{3}{4}\%$ for 3 yr. 7 mo. 19 da.

\$2874.35
.218⅙
.47905+
22.99480
28.7435
574.870
6) \$627.08735
\$104.5145+
3¾
313.5435
78.3859
\$391.9294
\$391.93 *Ans.*

6% for 1 year is for 3 years	.18
½% for 1 month is for 7 months	.035
$\frac{1}{60}\%$ for 1 day is for 19 days	.003⅙
6% for 3 yr. 7 mo. 19 da. is	.218⅙

Multiplying the principal by .218⅙ gives the interest at 6% for 3 yr. 7 mo. 19 da.

Dividing this product by 6 gives the interest at 1%. Multiplying the quotient by 3¾ gives the interest at 3¾%.

To find the interest at 6 per cent, multiply the principal by 6 times the number of years and ½ the number of months as hundredths, together with ⅙ the number of days as thousandths.

510. Written Exercises.

Find the interest at 6% on:

1. \$1428 for 1 yr. 4 mo. 6 da.
2. \$372.50 for 2 yr. 6 mo. 24 da.
3. \$1875 for 3 yr. 9 mo. 18 da.
4. \$240 for 4 yr. 7 mo. 15 da.
5. \$92.75 for 5 yr. 4 mo. 8 da.
6. \$817.80 for 10 mo. 19 da.

The interest at 6% plus ⅙ of itself gives the interest at 7%.
The interest at 6% minus ⅙ of itself gives the interest at 5%.
The interest at 6% plus ⅓ of itself gives the interest at 8%.
The interest at 6% minus ⅓ of itself gives the interest at 4%.
The interest at 6% plus ¼ of itself gives the interest at 7½%.
The interest at 6% minus ¼ of itself gives the interest at 4½%.

511. Written Exercises.

Find the amount:

1. $1875.25 for 3 yr. 5 mo. 15 da., at $4\frac{1}{2}\%$.
2. $487.50 for 1 yr. 10 mo. 25 da., at 6%.
3. $1206.84 for 2 yr. 1 mo. 16 da., at 5%.
4. $595.00 for 7 yr. 7 mo. 7 da., at 7%.
5. $763.25 for 8 mo. 11 da., at 4%.
6. $685.70 for 19 mo. 5 da., at $5\frac{1}{2}\%$.
7. $1563.00 for 3 mo. 20 da., at 5%.
8. $998.45 for 87 da., at $4\frac{1}{4}\%$.
9. $2575.50 for 149 da., at 3%.
10. $693.27 for 214 da., at $2\frac{1}{2}\%$.

Find the principal, rate, or time:

11. Principal, $240; interest, $32.04; time, 2 yr. 11 mo. 18 da. Rate?

12. Rate, 6%; amount, $717.40; time, 3 yr. 3 mo. 4 da. Principal?

13. Principal, $360; rate, 3%; interest, $48.87. Time?

14. Principal, $288; rate, $2\frac{1}{2}\%$; amount, $307.22. Time?

15. Rate, 6%; interest, $13.10; time, 4 mo. 11 da. Principal?

16. Principal, $270; amount, $273.27; time, 3 mo. 19 da. Rate?

17. Rate, $4\frac{1}{2}\%$; interest, $25.11; principal, $360. Time?

18. Interest, $50.22; time, 3 yr. 1 mo. 6 da.; rate, $4\frac{1}{2}\%$. Amount?

REVIEW OF BANK DISCOUNT.

512. Written Exercises.

Find face of note, term of discount, rate, discount, or proceeds:

By the *term* is meant the number of days the note has to run, including grace, if any.

1. Face, $600; discount, $6.30; rate, 6%. Term?
2. Term, 33 days; proceeds, $397.80; rate, 6%. Face?
3. Term, 90 days; face, $300; rate, 6%. Proceeds?
4. Term, 21 days; face, $600; discount, $2.45. Rate?
5. Term, 4 months; face, $200; rate, 6%. Discount?
6. Term, 132 days; proceeds, $2689.50; rate, 6%. Face?
7. Face, $150; proceeds, $147.75; rate, 6%. Term?
8. Face, $1650; discount, $4.95; rate, 6%. Term?
9. Term, 69 days; proceeds, $469.30; rate, 6%. Face?

EXACT INTEREST.

Exact interest is used by the United States Government in its calculations. 365 days are taken to the year.

513. Written Exercises.

1. Find the exact interest of $280 from April 14 to Sept. 6 at 4%.

Time, 145 days. *Ans.* $\$280 \times \frac{4}{100} \times \frac{145}{365}$.

2. Find the exact interest on $76.65 from March 4 to Dec. 15 at 6 per cent.
3. On $384 at $7\frac{3}{10}$ per cent for 75 days.
4. On $438 at 5% from Jan. 1 to March 15.
5. On $109.50 at $4\frac{1}{2}$% for 87 days.
6. On $847.60 at 5% from April 29 to Sept. 20.
7. $584 at $3\frac{3}{4}$% from May 16 to Dec. 1.

Unless "exact" or "accurate" interest is specified, use 360 days to the year.

MISCELLANEOUS.

514. Oral Review Problems.

1. A has 96 sheep; B has 28 sheep more than A. How many sheep have both?

2. There are 56 pupils in one class, 48 in a second class, and 52 in a third class. How many pupils are there in the three classes?

3. March 29 is what day of the year 1904?

4. How far is a man from his starting-point, if he travels due east 150 miles, due west 23 miles, due east again 48 miles?

5. A body falls 16 feet in the first second, three times as far in the second second, five times as far in the third second. How far does it fall in three seconds?

6. The base of a right-angled triangle is 12 feet, the perpendicular is 16 feet. What is the hypotenuse?

7. At $35 per month, what will be the rent of a house for 16 months?

8. A field containing 169 square rods is 13 rods long. What is the perimeter?

9. 25 packages of sugar weigh together $87\frac{1}{2}$ pounds. How many pounds are there in each?

10. At 45 miles per hour, how many hours, minutes, and seconds will it take a train to go 230 miles?

11. How many years have elapsed since the invention of gunpowder, 1356?

12. What profit is made on an article bought for $175, less 12%, and sold for $200?

13. How many square rods in a field 71 rods long, 81 rods wide?

14. Assuming a kilo to be $2\frac{1}{5}$ pounds, how many kilos will be equal to 143 pounds?

15. A degree of longitude in latitude 45° is about 70% of the length of a degree on the equator. Calling the latter length 69 miles, how long is a degree of longitude in latitude 45°?

16. At $44 per acre, how much land can be bought for $968?

17. A number of marbles divided among 29 boys gives each 16 marbles, and leaves a remainder of 26. How many marbles are there?

18. What is the monthly salary of a clerk who receives $1500 per year?

19. How many revolutions in a mile, 5280 feet, are made by a locomotive wheel 16 feet in circumference?

20. What is the perimeter of a lot 49 feet wide, 87 feet long?

21. How many bricks 8 inches by 4 inches by 2 inches would make a cubic foot?

22. 13 is one factor of 1001. Find the other two prime factors.

23. What are the three equal factors of 343?

24. What is the square root of 1225?

25. At $4\frac{1}{2}$ miles per hour, how long will it take a man to walk $37\frac{1}{2}$ miles?

26. What will be the cost of 9 dozen hats at $\$1.33\frac{1}{3}$ each?

27. Paid 92¢ for coffee, 48¢ for butter, and 18¢ for lard. How much was my bill?

28. I had $150. Spent $23 for a suit of clothes and $48 for tools. How much was left?

29. What is the area of a field 36 yards by 31 yards?

30. 600 hours equal how many days?

31. What is the cost of a cow if I pay $630 for 15?

32. How many ounces in $29\frac{1}{2}$ pounds?

33. $109\frac{1}{4}$ pounds of sugar are divided among 4 people. What is the share of each?

34. At $1\frac{9}{10}¢$ per pound, how many pounds of iron can I get for $5.70?

35. What is the cost of 51 tons iron at $17 per ton?

36. What will be the average age of 9 boys, each 12 years old, and 6 boys, each 10 years old?

37. At 42 miles per hour, how long will it take a train to go 882 miles?

38. At 25¢ per hour, what will a man earn in 18 days of 10 hours?

39. What will be the net price of an article whose catalogue price is $20.00, the discount being 90 and 10%?

40. A man had $181 in bank. What will be his balance after taking out $47 and $33?

41. How many feet in 14 rods?

42. 77 yards are how many rods?

43. How many square yards are there in a floor $10\frac{2}{3}$ yards long and $6\frac{1}{2}$ yards wide?

44. What is the cost of 372 eggs at 15¢ per dozen?

45. A man owns 3 farms containing 65 acres, 86 acres, and 98 acres, respectively. How many acres does he own?

46. What is the area of a piece of glass measuring $8\frac{1}{2}$ by $6\frac{1}{4}$ inches?

47. What is the value in U. S. money of 50 marks at $23\frac{8}{10}$ cents?

48. How many francs will a calf cost, if 18 are worth 630 francs?

49. A man spends $1740 per year. What is the average amount spent per month?

50. What would 51 pounds of butter cost at $33\frac{1}{3}¢$ a pound?

51. Mrs. Allen bought 7 chairs at $4 apiece, 2 tables at $9 apiece, and a carpet for $33. She paid two $50 bills. How much change was due her?

52. In what time will any sum of money double itself, at 6%?

53. Find the sum of the prime numbers as far as 12.

54. Interest of $1234, for 30 days, at 6%?

55. Interest of $1234, for 6 months, at 4%?

56. Oil is worth $37\frac{1}{2}¢$ a pint. How many pints can be bought for $6?

57. Sold oranges for $\frac{1}{2}¢$ apiece, gaining 50%. How much did they cost apiece?

58. What will be the cost of 1 pk. 1 qt. 1 pt. of nuts, at $10¢$ per quart?

59. What is the value of an acre of land, at $10¢$ per square foot?

60. 3 desks are bought at $10 each, and sold for $45. Find the rate of gain.

61. A wheelman sells his old bicycle for $25, and loses $16\frac{2}{3}\%$. How much did it cost him?

62. How much does an agent get for buying 5 bales of goods at $400 each, if he receives 3% for his services?

63. 10% of 200 is $\frac{2}{3}$ of what number?

64. How old, December 1, 1903, was a boy born November 25, 1889?

65. A man has $1000 in bank. What will remain after he has taken out $478?

66. How many hours in the month of January?

67. In how many years, months, and days will $100 amount to $111, at 5%, simple interest?

68. What will 5 tons of granulated sugar cost, at $6\frac{1}{4}$¢ per pound?

69. What is the interest of $50, for 3 yr. 7 mo. 12 da., at 6%?

70. A farmer makes 675 gallons of cider. He has but 12 barrels, each of 45 gallons' capacity, to store it in. How many more such barrels does he need?

71. What will be the cost of 36 yards of cloth, at $2.75 per yard?

72. Add 379 and 297.

73. What is the bank discount on a sixty-days note for $400, at 6%?

74. Change $\frac{3}{11}$ to a decimal of three places.

75. How much wood in three piles containing, respectively, $\frac{1}{2}$ of a cord, $\frac{1}{3}$ of a cord, and $\frac{1}{4}$ of a cord?

76. What is the percentage of gain in case of railroad stock bought for $80 per share, and sold for $90 per share?

77. A dealer sold flour at a profit of 50¢ a barrel, and gained 10%. What was the cost?

78. At 10¢ a quart, what are 3 bu. 1 pk. 5 qt. of chestnuts worth?

79. How many yards in 288 inches?

80. What decimal of a number is $\frac{2}{5}$ per cent of it?

81. If a broker buys for me 5 shares of railroad stock whose par value is $100, what is his brokerage at $\frac{1}{4}$%?

82. If I sell 10 shares of railroad stock for $1090, and gain 9% on the cost, what was the cost?

83. What is the interest of $660, for 3 months, at 4%?

84. What per cent does a merchant lose by selling goods at $\frac{4}{5}$ of their cost?

85. What principal at 6% simple interest will gain $36 in 1 year and 6 months?

86. What per cent is gained on goods sold at double the cost?

87. What is 8% of 50 bushels?

88. $3000 is $11\frac{1}{9}$% of my property. How much am I worth?

89. What is the interest on $700, for 15 days, at 6%?

90. Bank discount on a 65-days note for $1000, discounted at date?

91. At what rate will $2 gain $20 in 5 years?

92. A capitalist wishes to realize 5% on money invested in stock. What must be the annual dividend on stock costing 300, in order to produce this rate?

93. What will be the taxes on property assessed at $25,000, the rate being $16 per $1000?

94. Find the compound interest on $1000, for two years, at five per cent, interest compounded annually.

95. What will be the net cost of an article marked $8, on which a discount of 50, 25, and 10% is allowed?

96. Find the "list" price of an article sold for $10 after a discount of 50 and 50 per cent had been deducted.

97. Paid 90¢ for an article. The discount is 25 and 25 per cent. What is the list price?

98. One boy can do a certain piece of work in 2 hours, a second boy requires 3 hours, a third needs 6 hours. How long will it take the three working together?

99. Sold a cow for $60, losing 25%. What was the loss?

100. Sold a cow for $60, gaining 25%. What was the gain?

101. Sold two horses at $240 apiece. On one I gained 20%, on the other I lost 20%. Did I gain or lose on both, and how much?

102. What is the interest of $1500, for 60 days, at 6%?

103. How many years will it take $20 to gain $20 at 5 per cent simple interest?

104. John has $60, James has $80. James has what per cent more money than John? John has what per cent less money than James?

105. $\frac{2}{3}$ is what per cent of $\frac{1}{2}$? $\frac{1}{2}$ is what per cent of $\frac{2}{3}$?

106. Two men working together can finish a piece of work in 8 days; one can do it in 12 days. How long would the other take to do the work?

107. How many yards of cloth at $3.75 per yard can be bought for $90?

108. A puts $600 into business; B, $400; the profits are $500. What is the share of each?

109. Two boys hire a camera for 26 weeks, paying $5.20. How much should be paid by the boy that uses it 12 weeks?

110. New Orleans is 90° west of Greenwich. When it is 2 P.M. at the latter place, what is the time at New Orleans?

111. Find the discount, at 6%, on a note for $300, that has 48 days to run.

112. What will be the cost of 84 yards of muslin at 49¢ a yard?

113. Two men hire a pasture for $84. One puts in twice as many head of cattle as the other. What should each pay?

114. A base-ball club won 17 games, and lost 13 games. What per cent of its games did it win?

115. What per cent of 4 is 64?

116. $2\frac{2}{3}$ is what per cent of $3\frac{1}{5}$?

117. How many acres in a rectangular farm 1 mile long, $\frac{7}{8}$ mile wide?

118. What per cent of the "list" price is paid by a buyer who receives a discount of 20 and 10 per cent?

119. A tank is filled by two pipes, one of which can fill it in 6 hours, and the other in 8. How long will it take both together to fill the tank?

120. Find the interest on $80, for 72 days, at 6%.

121. A man sold a wagon for $420, which was 16% less than it cost. How much did he lose?

122. A kilo is 2.2046 lb. How many pounds in 1000 kilos?

515. Written Review Problems.

1. What number subtracted 88 times from 80.005 will leave .013 as a remainder?

2. At what price must an article that cost $30 be marked so that after deducting 40% from the marked price, 30% profit may be realized?

3. Write a ninety-days promissory note for which you should get $240 at the bank, discount being 6%.

4. If a horse dealer buys a span of horses at 10 per cent less than their value, and sells them at 10 per cent more than their value, what per cent does he make?

5. If a boy buys peaches at the rate of 5 for 2 cents, and sells them at the rate of 4 for 3 cents, how many must he buy and sell to gain $4.20?

6. What is the difference between the compound interest on $5000, for 3 years, at 5%, and on $10,000, for $1\frac{1}{2}$ years, at the same rate?

7. A can do a piece of work in 27 days, and B in 15 days; A works at it alone for 12 days, B then works alone for 5 days, then C finishes the work in 4 days. In what time could C have done the work by himself?

8. A room is 15 feet long, 10 feet broad, and 9 feet 9 inches high. Find the cost of painting the walls and the ceiling, at 1*s.* 9*d.* a square yard.

9. What is the value of a pile of wood 40 feet long, 4 feet wide, and 5 feet high, at $5.30 a cord?

10. By buying a cargo of coal at $6 per ton, and selling it at $8 a ton, I gained $198. How much did I pay for it?

11. Make out a receipted bill for the following: 325 yards of silk at $2.25 per yard; 296 yards of lace at $1.50 per yard; 480 yards of ribbon at $0.50 per yard; 45 dozen pairs of gloves at $15 per dozen pairs.

12. My dividend is $8\frac{3}{7}$, quotient $9\frac{4}{5}$. What is the divisor?

13. I gave away $\frac{1}{3}$ and $\frac{2}{5}$ of $4\frac{1}{2}$ bushels of chestnuts. What % was left?

14. The perimeter of a square field is 16 rods. What is the field worth, at $8\frac{1}{3}¢$ a square foot?

15. A broker's bill for cotton at $4\frac{7}{8}¢$ per pound and his commission for buying at $2\frac{1}{2}\%$ was $1998.75. How many bales of 400 pounds each did he buy, and what was his commission?

16. I sold 80 yards of broadcloth for $240, thereby losing 20% on the cost. For what should I have sold it per yard to have gained 15% on the cost?

17. A man bought 60 casks, of 65 gallons each, for $1542; 80 gallons leaked out. For what must he sell the remainder per gallon to gain $12\frac{1}{2}\%$ on the cost?

18. Each of two men sold his horse for $180. One made 20%, the other lost 20% on the cost. Cost of each horse?

19. A man agrees to dig a cellar 30 feet long, 24 feet wide, and 6 feet deep. What % of the work is to be done when he has removed 144 cubic yards?

20. A man bought 672 yards of cloth at $1.25 a yard. He sold it immediately for $2.25 a yard, receiving in payment a 60-days note for the amount, which he had discounted at a bank at 7%. How much money did he make?

21. What will it cost to fill in a street 55 feet wide, 600 feet long, and $5\frac{1}{2}$ feet below grade, at 40¢ a cubic yard?

22. The quotient arising from the division of 6985.473 by a certain number is 51, and the remainder is 68.853. What is the divisor?

23. What is the value of the following?

$$\frac{3\frac{3}{16}+4\frac{4}{5}-6\frac{5}{8}}{2\frac{4}{7}+1\frac{3}{4}-3\frac{1}{2}} \div \frac{\frac{3}{8}\times\frac{7}{10}\times\frac{2}{9}}{\frac{8}{7}\times\frac{3}{4}\times\frac{2}{5}}.$$

24. In going 1 mi. 94 rd. 2 yd. 1 ft., a carriage wheel makes 526 revolutions. What is the circumference of the wheel?

25. On a note dated Oct. 16, 1903, for $2650, with interest at 6 per cent, the following payments were made: Jan. 28, 1904, $575; May 22, 1904, $25; and Aug. 4, 1904, $948. What was due Nov. 25, 1904?

26. A grocer pays 18¢ per pound for coffee and roasts it, the coffee losing 10 per cent of its weight in the process. What must he charge per pound for the roasted coffee in order to make a profit of 20 per cent, allowing 4 per cent for bad debts?

NOTE. — 96% of the price he receives per pound must be 20% more than the rate of 18¢ for $\frac{9}{10}$ lb.

27. A merchant imported from Bremen 32 pieces of linen of 32 yards each, on which he paid for the duties, at 24 per cent, $122.16, and other charges to the amount of $40.96. What was the invoice value per yard, and the cost per yard after duties and charges were paid?

28. A garrison of 1200 men is provisioned for 100 days. At the end of 30 days, 600 men are withdrawn, and at the end of 60 days, 900 men are added. How long will the provisions last?

29. What will be the result, if $\frac{1}{5}$ of $\frac{3}{4}$ of $3\frac{1}{8}$ be multiplied by $\frac{1}{2}$ of itself, and the product be divided by $\frac{1}{2}$?

30. A collector of internal revenue deposited in the treasury $762,742.50, retaining $2\frac{1}{2}$ per cent of the amount collected. What amount did he collect?

31. What is the duty on 25 tons 2 cwt. 3 qr. of iron at $40 per ton? (1 ton = 2240 lb.)

32. An importer sold a part of a cargo of tea at 30 cents a pound and made a profit of 20 per cent. What per cent did he make on the remainder of the cargo, which he sold at 40 cents a pound?

33. Divide $4.14 among Thomas, Richard, and Henry in such a way that Henry shall receive 3 cents for every 5 cents that Thomas gets, and Richard shall receive 2 cents for every 5 cents that Henry gets.

34. Reduce 272 liquid quarts to dry quarts.

35. A pipe discharging 3 gallons 1 pint a minute fills a tub in 4 minutes 20 seconds. Another pipe discharges 83 quarts a minute. If both pipes discharge together into the tub, how long will they take to fill it?

36. William Wilson sold goods to the amount of $1000. One-half of his sales showed a profit of 25 per cent on the cost, and the remaining half a loss of $16\frac{2}{3}$ per cent on the cost. Required the total cost of the goods.

37. If I sell $\frac{2}{3}$ of my farm for $\frac{3}{4}$ of what the farm cost me, what is my gain per cent?

38. Which is the higher rate of freight on wheat, $.16 per hundred or $.10 per bushel (60 lb.), and what per cent?

39. Write in words:

(*a*) .267; (*b*) 200.067; (*c*) $\frac{260}{7000}$; (*d*) $200\frac{60}{7000}$.

40. If 40 per cent of the selling price of an article is profit, what is the per cent of gain on the cost?

41. What number added to $4\frac{1}{2}$ times itself will equal $60\frac{1}{2}$?

42. Divide $\frac{1}{3}$ by $.00003\frac{1}{3}$.

43. Reduce to lowest terms (*a*) $\frac{1261}{2087}$; (*b*) $\frac{357}{4855}$.

44. If 4 men eat 64 pounds of bread in 2 weeks, how many pounds will 16 men eat in 7 weeks at the same rate?

45. Divide .75 of $17\frac{5}{8}$ by $\frac{4}{5}$ of .035.

46. Find the cost of 3846 pounds of hay at $15 per ton.

47. Find the cost of plastering the walls and the ceiling of a hall 72 feet long, 50 feet wide, and 22 feet high, at $18\frac{3}{4}$ cents a square yard, allowing 972 square feet for openings and baseboards.

48. A certain quantity of paper will make 4000 copies of an octavo book (8 pages to the sheet). How many copies of a 12mo book (12 pages to the sheet) will the same paper make?

49. Find the diagonal of a square park containing 20 acres.

50 BANGOR, MAINE, June 24, 1904.

On demand, I promise to pay Joseph I. Totten, or order, Two Thousand Five Hundred Fifteen Dollars, with interest, value received.

$2515\frac{00}{100}$. CHARLES HETTESHEIMER.

$1541.01 was paid Jan. 1, 1905. Find the amount due Aug. 15, 1905.

51. How much does it cost annually to insure the "Celtic" for $1,525,000, if $2\frac{1}{4}\%$ is paid for the insurance?

52. $150 is paid an agent for purchasing 1200 barrels of flour on a commission of $2\frac{1}{2}\%$. How much was paid per barrel for the flour?

53. An agent received $2562.50 for purchasing land at $62.50 per acre, and his commission of $2\frac{1}{2}\%$. How many acres did he buy?

54. Reduce the fraction $\dfrac{8\frac{1}{2} \times \frac{7}{13}}{\frac{1}{12} \text{ of } \frac{3\frac{1}{2}}{\frac{4}{5}}} \div \dfrac{\frac{1}{3}}{\frac{3}{4} \text{ of } 7\frac{7}{11}}$.

55. Divide $\frac{7}{8}$ by 2.5, to the quotient add the divisor, and from that sum subtract the dividend. Give the fractional part of the answer in a decimal.

56. If the interest on $300 for 1 yr. 8 mo. is $36, find what would be the interest on $212.50 for 3 yr. 4 mo. 24 da. at the same rate.

57. Reduce .0468 T. to a compound number.

58. Find the prime factors of 20,930.

59. A man paid $999 for the rent of a house from June 29, 1903, to May 5, 1905. What was the rent per year?

60. What per cent of 3 lb. 7 oz. is 7 lb. 9 oz.?

61. At 50 cents per running yard, what will be the cost of fencing a square field containing 10 acres?

62. At the rate of 20 problems an hour for A, and 15 in 55 minutes for B, in what time can both together solve 100 problems?

63. Find the entire surface of a cube whose edge measures 15 inches.

64. A dealer buys books at $1.50 each, less $33\frac{1}{3}$ and 10 per cent. At what price per copy must he sell them to gain $43\frac{1}{8}$ per cent?

65. Abraham Lincoln died at the age of 56 yr. 2 mo. 3 da., after serving as President 4 yr. 1 mo. 11 da. Give the date of his birth, the date of his inauguration being March 4, 1861.

66. A dealer buys 150 barrels of flour. He sells one-third of it at $4.50 per barrel, losing 10 per cent. The remainder he sells at a profit of 6 per cent. What is his net gain or loss?

67. Sixty per cent of $66\frac{2}{3}$ per cent of a number equals 810. What is the number?

68. A ladder 40 feet long is so placed in a street, that without being moved at the foot, it will reach a window on one side 33 feet, and on the other side 21 feet from the ground. What is the breadth of the street?

69. Four men hired a coach for $13, to convey them to their respective homes, which were at distances from the place of starting as follows: A's 16 miles, B's 24 miles, C's 28 miles, and D's 36 miles. What ought each to pay?

70. What is a pile of wood 8 feet long, 7 feet wide, and 5 feet high worth, at $4.50 per cord?

71. When bank stock sells at a discount of $7\frac{1}{2}$ per cent, what amount of stock, at par value, will $3700 purchase?

72. The pound sterling is worth $4.8665. How much U. S. coin would it require to pay a debt of £780 18*s.* 11*d.*?

73. A merchant imported 120 tons of English iron, costing $1\frac{1}{2}$ pence per pound, on which he paid a duty of 20 per cent. The freight was 5 shillings sterling per ton. What was the total cost in U. S. currency? (1 ton = 2240 pounds. £1 = $4.8665.)

74. How many rods of fence are required to enclose a square lot whose area is 5184 square feet?

75. Property worth $6000 is insured for $\frac{3}{4}$ of its value, at $\frac{3}{4}$ of one per cent. What will be the loss, including premium, in case of total destruction by fire?

76. How many acres of land, in the form of a square, may be enclosed by 160 rods of fence?

77. Find the square root of .441 correct to two decimal places.

78. Reduce 17 lb. 10 oz. Avoirdupois weight to pounds, ounces, pennyweights, and grains, troy weight. (1 pound Avoirdupois = 7000 Troy grains.)

79. Reduce $\frac{1}{2}\frac{5}{7}\frac{1}{5}\frac{3}{9}$ to its lowest terms.

80. Find the solid contents of a cube, the area of one face of which is 256 square feet.

81. A car contains 21,643 pounds of wheat. Find the value of the load at 92 ¢ per bushel of 60 pounds.

82. Find the area of a triangle whose base is 22 ft. 8 in., and altitude 19 ft. 9 in.

83. The list price of a certain stove is $ 38, and the retail dealer is allowed commercial discounts of 20 per cent, 5 per cent, and 3 per cent. What price does he pay for the stove?

84. If a ton of coal lasts a family 21 days, what will be the cost of coal used by it from Oct. 17, 1904, to April 25, 1905, exclusive of either day named, at $ 4.50 per ton?

85. Find the cost of a pile of 4-foot wood, 27 feet long and 6 feet high, at $ 5.50 per cord.

86. How many rods of fence will be required to enclose a field in the form of a right-angled triangle, whose area is $13\frac{1}{2}$ acres and whose base measures 48 rods?

87. What is the balance of a bill of $ 64.50, after two discounts have been made; the first of 20% on the $ 64.50, the other of 5% on what then remained?

88. There was shipped to Liverpool from New York in one week $ 6,870,205 in specie. What amount of English currency could be bought with it? (£ 1 = $ 4.8665.)

89. What is the freight on 9860 pounds iron at $ 1.75 per ton?

90. What is the value of 10 lb. 7 oz. 16 pwt. of gold at $.75 a pennyweight?

91. The dividend is 6171, the quotient 17, the remainder 102. What is the divisor?

92. Divide the L. C. M. of 132 and 156 by their G. C. D.

93. The product of three numbers is .0728; one of them is 1.3, another .07. Find the third.

94. If 5 men can make 38 rd. 5 yd. of fence in a day, how much can they build in 30 days?

95. The distance from New York to New Haven being 73 mi. 8 rd., at what rate does a train run per hour to cover the distance in 2 hr. 10 min.?

96. Reduce 4 da. 4 hr. 48 min. to the decimal of a week.

97. After 4 per cent of a flock of sheep had been killed by dogs, and 68 had been sold to a butcher, four-sevenths of the original flock were left. Required the number of sheep in the flock at first.

98. Six men bought a ship worth $45,268, for which A paid $\frac{1}{4}$ of the whole, B $\frac{1}{5}$, and the others paid the remainder equally. How much did each of the latter pay?

99. A man agrees to dig a cellar 30 feet long, 24 feet wide, and 6 feet deep. What per cent of the work has he done when he has removed 16 cubic yards?

100. How many boards 16 feet long, and 4 inches wide, are required to floor a room 48 feet long, and 32 feet wide?

101. How much walking does a man save by crossing diagonally a field 28 rods long, and 21 rods wide, instead of going along the end and the side?

102. In order to have an annual income of $2500, what sum must be invested at 5%?

103. At $2 a rod, what is the difference in the cost of fencing a lot of land 20 rods square, and another lot containing the same area which is 40 rods long?

104. If a man owning 45% of a steamboat sells $\frac{1}{8}$ of his share for $5860, what is the value of the whole boat?

105. A farmer having 6 bu. 8 qt. of cranberries lost by decay 7 pk. 7 qt. What % had he left?

106. Sold tea for 114% of its cost, and made a profit of 7¢ a pound. Find selling price.

107. In $\frac{2}{3}$ of an acre of land how many building lots, each 60 feet by 121 feet?

108. I bought a store for a certain sum, and after paying a tax of $2\frac{1}{2}$% on the cost and $\frac{1}{2}$% more for insurance I sold it for $7828, which exactly covered the cost, tax, and insurance. What was the cost?

109. Parker P. Simmons, of Vermont, sent to Nostrand Bros. of Boston, to be sold on commission, the following goods: 25 tons of hay, 2 tons of butter, 1500 pounds of maple sugar, 75 gallons maple syrup. Nostrand Bros. sell the hay at $18 a ton, the butter at 20¢ a pound, the sugar at 7¢ a pound, the syrup at 90¢ a gallon.

Nostrand Bros. charge 2% commission. How much do they send to Parker P. Simmons?

110. How much will a granite block weigh which is 7 feet long, 2 ft. 6 in. wide, 3 ft. 4 in. high? (12 cubic feet of granite weigh a ton.)

111. A coal dealer bought 350 tons of coal, weighing 2240 pounds each, at $3.50 a ton. He sold the coal at $4.25 a ton, each ton weighing 2000 pounds. What was his profit?

112. Mrs. Burns buys 40 yards of carpet $\frac{3}{4}$ of a yard wide. She uses 10% of it for a rug, and the remainder to carpet a floor. How many square yards does she use for the floor?

113. Mr. Burns sold his carriage for $224, which was $\frac{8}{7}$ of its cost. What per cent would he have gained if he had sold it for $210?

114. What is the difference between four thousand nine and seven hundred eighty-six ten-thousandths, and four hundred thousand nine and seven hundred eighty-six millionths?

115. Discover a fraction which, multiplied by $\frac{4}{5}$, equals $\frac{2}{3}$.

116. What % of $\frac{1}{2}$ of $\frac{5}{4}$ of $\frac{8}{3}$ is $\frac{1}{3}$?

117. How many inches in $\frac{1}{16}$ of a mile?

118. Bought a horse for \$90, and sold him for \$95. What per cent of gain? Bought another horse for \$95, and sold him for \$90. What per cent was lost?

119. Bought land at \$62.50 per acre, and sold it again at \$75 per acre, thereby making \$8468.75. How many acres were bought?

120. Two ships sail from the same port; one goes due north 128 miles, and the other due east 72 miles. How far are the ships from each other? Illustrate.

121. B and C, trading together, find their stock to be worth \$3500, of which C owns \$2100. They have gained 40% on their first capital. What did each put in?

122. A general wished to remove 80,000 pounds of provisions from a fortress in 9 days. It was found that in 6 days 18 men had carried away but 18 tons. How many men would be required to carry away the remainder in 3 days?

123. A schoolroom is 40 feet long, 30 feet wide, and 14 feet high. Find the difference between the length of a diagonal drawn on the floor and one drawn from the floor to the ceiling.

124. Find the solid contents and the surface of a sphere 12 inches in diameter.

125. The number of copies in the first edition of the "Lady of the Lake" was 2050, and was to the number in the second edition as 41 to 69. Find the number in the second edition.

126. Find the proceeds of the following note:

$\$1050\frac{00}{100}$. CHICAGO, Feb. 13, 1905.

Six months after date I promise to pay to the order of John G. Agar One Thousand Fifty Dollars, with interest at 6 per cent. HENRY R. M. COOK.

Discounted at 8 per cent, May 13.

127. A can do $\frac{1}{3}$ of a piece of work in 4 days, and B can do $\frac{1}{4}$ of it in 5 days. In what time can they do the whole work together?

128. A square is inscribed in a circle whose diameter is 84 inches. Find the area of the four segments of the circle outside of the square.

129. Find the difference between the volume of a cylinder whose diameter and height are 12 inches, and the volume of a sphere whose diameter is the same.

130. A certain cistern can be filled by one pipe in 10 hours, by another in 6 hours, and can be emptied by a third in 5 hours. In how many hours can it be filled if all three pipes are opened at once?

131. Two men start from two towns 105 miles apart and walk toward each other. They meet at the end of 15 hours. The first has travelled 3 miles per hour. At what rate has the second travelled?

132. If a cipher is added at the right of the decimal, what effect has this on the value of the decimal? Explain the reason.

133. What is the easiest method of dividing a decimal by 10?

134. If the numerator of a common fraction is divided by 3, what is the effect upon the value of the fraction?

135. If the denominator is divided by 3, what is the effect upon the value of the fraction?

136. What is the effect on the value of a decimal of moving the decimal point two places to the right? Explain the reason.

137. What is the exact interest on $400, from March 1 to December 17, at 5 per cent? (365 days to the year.)

138. In what time will a principal amount to $2\frac{1}{2}$ times itself, at 10 per cent?

139. A and B in partnership have together a capital of $7500, and gain $1200. A's share of the gain is $250. What is B's share of the gain? What is B's share of the capital?

140. The circumference of a circle is 15.708 feet. What is the radius of it?

141. The radius of a circle is 42. What is the circumference of it?

142. Find the entire surface of a cylinder 10 inches long, and 8 inches in diameter. Find the number of cubic inches in the same cylinder.

143. Explain the reason for multiplying the second and third terms together and dividing by the first term in solving an example in simple proportion.

144. Divide thirty-two hundred-millionths by sixty-four ten-thousandths.

145. A, B, and C gained by speculation $11,480, of which A's share was twice as much as C's, and B's five times as much as C's. How much did each gain?

146. A pole was broken 52 feet from the bottom, and it fell so that the end struck 39 feet from the foot. Required the length of the pole.

147. Sold a horse so that $\frac{4}{5}$ of the gain equalled $\frac{2}{15}$ of the cost. What was the gain per cent?

148. In what time will the interest on £57 1*s.* 8*d.* amount to £2 11*s.* $4\frac{1}{2}$*d.* at $7\frac{1}{2}$ per cent per annum?

CHAPTER VII.

ALGEBRAIC EQUATIONS.

ONE UNKNOWN QUANTITY.

516. A number increased by 12 equals 16.

This may be written, $x + 12 = 16$.

The second way is shorter. Here x stands for the number.

517. Preliminary Exercises.

Tell what x may stand for, and write in a short way each of the following:

1. A number increased by 5 equals 7.
2. 6 is added to a number. The sum is 9.
3. 4 subtracted from a number leaves 1.
4. 12 diminished by a number has 8 for a remainder.
5. A number is subtracted from 10. The remainder is 3.
6. 10 is subtracted from a number. The remainder is 3.
7. The number of years of John's age added to 3 years equals 15 years.
8. In two years Mary will be 11 years old.
9. 5 years ago Thomas was 8 years old.
10. If William should add 5 marbles to the number he now has, he would have 15 marbles.
11. If Kate spends 10 cents, she will have 15 cents left.
12. When paying for a top, Henry received 7 cents change from 10 cents.

13. A ball and a bat together cost 40 cents. The bat cost 15 cents.

14. A watch cost Mr. Smith $60. He bought the case and the works separately. The works cost $20.

15. The weight of a loaded wagon is 3200 pounds. The load weighs 2000 pounds.

518. Sight Exercises.

If $x + 7 = 9$,
then $x = 2$,
because $2 + 7 = 9$.

Find the value of x:

1. $x + 6 = 9$.
2. $x - 4 = 1$.
3. $x + 5 = 7$.
4. $10 - x = 3$.
5. $x - 10 = 3$.
6. $12 - x = 8$.
7. $x + 2 = 11$.
8. $x + 3 = 15$.
9. $x + 5 = 15$.
10. $x + 5 = 8$.
11. $x - 7 = 10$.
12. $x - 10 = 15$.
13. $x + 20 = 60$.
14. $x + 15 = 40$.
15. $x + 2000 = 3200$.
16. $x + \frac{1}{2} = 1$.

COEFFICIENTS.

519. (1) $3x$ means 3 times x.
(2) $2\frac{1}{2}a$ means $2\frac{1}{2}$ times a.
(3) ax means a times x.
In (1), 3 is the coefficient of x.
In (2), $2\frac{1}{2}$ is the coefficient of a.
In (3), a is the coefficient of x.

Notice that the coefficient and its letter are written side by side. Is there any sign between them? What sign is understood? What is a coefficient?

520. Written Exercises.

Write in a short way, and tell what x stands for.

1. 8 times a number = 64.

2. A butcher receives 63 cents for a piece of meat at 9 cents a pound.

3. $2\frac{1}{2}$ yards of muslin cost 25 cents.

4. A lady paid 40 cents for 3 spools of black silk and 2 spools of white silk at the same price per spool.

5. A man worked by the day 10 days on my barn and 8 days on my house. For all this work he received \$ 36.

6. A man spent $\frac{1}{4}$ of his week's wages for a pair of boots. The boots cost him \$ 3.

7. 11 times a number less 2 times the number is 27.

8. John's money is in pennies and nickels. He has the same number of each. He has 42 cents.

521. Sight Exercises.

If $10x - 7x = 18$,

then $3x = 18$,

and $x = 6$.

PROOF: $60 - 42 = 18$.

Give value of x at sight:

1. $8x = 64$.
2. $9x = 63$.
3. $3x + 2x = 40$.
4. $2\frac{1}{2}x = 25$.
5. $\frac{1}{4}x = 3$.
6. $10x + 8x = 36$.
7. $4x - 3x = 72$.
8. $11x - 2x = 27$.
9. $3x - 2x + 5x = 54$.
10. $10x + 8x - 4x = 42$.
11. $2x + 4x = 52 - 16$.
12. $3x + 4x = 30 - 9$.
13. $12x - 5x = 25 + 10$.
14. $6x + 6x = 16 + 8$.

522. Oral Exercises.

If $a = 2$, and if $b = 3$, and if $c = 7$.

1. $a + b = ?$
2. $b - a = ?$
3. $a + b + c = ?$
4. $c - a + b = ?$
5. $c - a - b = ?$
6. $ab = ?$
7. $ab + c = ?$
8. $ac - b = ?$
9. $abc = ?$
10. $? = 9.$
11. $? = 21.$
12. $? = 23.$
13. $? = 19.$
14. $? = 4.$

523. Written Problems.

1. A horse and a wagon cost together \$600. What is the price of each, if the wagon costs twice as much as the horse?

Let $x =$ cost of horse;

then $2x =$ cost of wagon.

Cost of both $= 2x + x = 600.$

$3x = 600.$

$x = 200.$

$2x = 400.$

Ans. Cost of horse, \$200; of wagon, \$400.

2. Divide 100 into two parts, one of which shall be four times as large as the other.

Let $x =$ one part;

then $4x =$ the other.

$x + 4x = 100.$

3. \$18,000 is divided among three children, the second of whom receives twice as much as the first, and the third of whom receives six times as much as the first. Required the share of each. $x,\ 2x,\ 6x.$

4. In a class of 54 pupils, there are twice as many boys as girls. How many are there of each?

5. The sum of two numbers is 78. One is five times as large as the other. What are the numbers?

6. 156 is equal to seven times a number added to five times the same number. Find the number.

7. The difference between three times a certain number and nine times the same number is 66. What is the number?

8. $27,000 is divided among three children, the second of whom receives twice as much as the first, and the third of whom receives three times as much as the second. What is the share of each?

9. The sum of two numbers is 72, and the greater is 5 times the other. What are the numbers?

10. John, Henry, and James have 54 marbles. Henry has twice as many as John, and James has as many as the other two. How many has each?

11. The sum of the ages of mother and daughter is 42 years. What is the age of each, if the mother's age is six times that of her daughter?

12. A man paid $96 for an equal number of hats and coats, paying $2 apiece for the former and $10 apiece for the latter. How many of each did he buy?

Let x = number of each,

then $2x$ = cost of hats,

$10x$ = cost of coats.

13. Divide 41 into four parts, the first being twice the second, the second three times the third, and the third four times the fourth. (Let x = the fourth.)

14. The sum of three numbers is 180. The first is double the second, and the third is three times as large as the sum of the other two. What are the numbers?

15. Mr. Smith paid 81 cents for sugar and flour, the same quantity of each. For the sugar he gave 5¢ per pound, and for the flour 4¢ per pound. How many pounds of each did he buy?

16. The length of a rectangular field is 24 rods, its breadth is x rods, its area is 456 square rods. Find the value of x.

17. It takes 340 feet of fence to enclose a square lot. What are the dimensions of the lot?

18. Mrs. B. divides $120 between her son and her daughter. She gives the latter twice as much as she gives the former. What is the share of each?

19. The earnings of a man and his son during January amounted to $175, both having worked the same number of days. The father's wages were $4 per day, and the son's wages were $3 per day. How many days did they work?

20. The sum of $240 is divided among four children, two boys and two girls. Find the share of each, if each girl's share is double that of each boy.

21. A man worked twice as many days as his son. Their combined earnings amounted to $165. Find the number of days each worked, if the father earned $4 per day and the son three-fourths as much per day.

22. A boy's bank contains 78¢ in dimes, nickels, and cents. There are twice as many nickels as there are dimes, and three times as many cents as there are nickels. How many are there of each?

23. I paid 75¢ more for a roll of 15-cent ribbon than I did for a roll of 12-cent ribbon of the same length. How many yards did each roll contain?

24. A rectangular field whose length is four times its breadth requires 250 rods of fence to enclose it. What are the dimensions of the field? (Make diagram.)

25. A girl paid 60 cents for a speller and a reader, the cost of the former being one-third that of the latter. Find the cost of each.

26. The sum of two numbers is 72, and the smaller is one-fifth of the other. What are the numbers?

Let x = smaller.

27. Mary, Susan, and Jane have 54 hickory nuts. Susan has one-half as many as Mary, and Jane has as many as the other two. How many has each?

Let x = number Susan has.

EQUATIONS.

524. An expression like $3x + 16 = 28$ is an *equation.*

$3x + 16$ is the first member of the equation.

28 is the second member of the equation.

What is the part of the equation to the left of the equality sign called? What is the second member of an equation? What sign is between the two members? What does this sign show about the value of the two members? What is an equation?

525. Written Exercises.

Suppose $a = 2$ and $b = 3$.

Complete the equations:

1. $ab + ? = 7.$
2. $a + b - ? = 4.$
3. $?a + b = 11.$
4. $ab = a + b + ?$
5. $ab = 12 - ?\,b.$
6. $ab + b = 5 + ?\,a.$
7. $17 - ab = 5a + ?$
8. $17 - ab - ? = 5a.$
9. $12 + ab = 2b + ?\,a.$
10. $12 + ab - ?\,a = 2b.$

CLEARING OF FRACTIONS.

526. Oral Exercises.

1. One-fifth of a number is 4. What is the number?

2. $\frac{1}{5}$ of a number is 8. What is $\frac{2}{5}$ of the number?

3. $\frac{1}{4}$ of a number is 12. What is the number?

4. $\frac{1}{4}$ of a number is 10. What is $\frac{3}{4}$ of the number?

5. If $\frac{3}{4}$ of a number is 30, what is the number?

6. One-half a number added to $\frac{1}{4}$ of the same number equals what fraction of the number?

7. One-half a number added to $\frac{1}{4}$ of the same number equals 30. What is the number?

8. One-third of a number + one-sixth of the number = what fraction of the number?

9. One-third of a number added to $\frac{1}{4}$ of the number = what fraction of the number?

10. $\frac{1}{3}x + \frac{1}{4}x =$ what fraction of x? $\frac{x}{3} + \frac{x}{4} = ?$

527. When $x = 32$, find the value of three-fourths of x; *i.e.*, $\frac{3x}{4}$.

When $\frac{3x}{4}$ ($3x$ divided by 4) $= 24$, what is the value of $3x$? Of x?

Find the value of y, when $\frac{y}{3} = 12$. Of $2y$, when $\frac{2y}{3} = 24$.

Given the equation $\frac{4z}{5} = 20$; by what whole number can we multiply the first member to get rid of the fraction? If we multiply one member of an equation by any number, what must we do to the second member in order to preserve the equality?

If equals are multiplied by equals the products are equal.

528. Sight Exercises.

Give values of x, y, z, etc.:

1. $\frac{x}{5}=4.$
2. $\frac{2y}{5}=8.$
3. $\frac{z}{4}=7.$
4. $\frac{3v}{4}=21.$
5. $\frac{w}{2}+\frac{w}{4}=12.$
6. $\frac{x}{2}+\frac{x}{3}=5.$
7. $\frac{y}{3}+\frac{y}{6}=10.$
8. $\frac{z}{3}+\frac{z}{4}=7.$
9. $\frac{v}{5}+\frac{v}{5}=8.$
10. $\frac{w}{3}+\frac{2w}{3}=32.$
11. $\frac{x}{4}+\frac{x}{5}=9.$
12. $\frac{x}{2}+\frac{2x}{5}=9.$
13. $\frac{x}{2}-\frac{x}{4}=2.$
14. $\frac{x}{3}-\frac{x}{12}=3.$
15. $\frac{x}{2}-\frac{x}{5}=6.$
16. $\frac{x}{2}-\frac{x}{7}=5.$

529. Written Exercises.

Find the value of the unknown quantity (x).

1. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=26.$

Multiplying by 12, we have $6x+4x+3x=312.$

2. $x+\frac{x}{2}+\frac{x}{3}=44.$

Multiply by 6. $6x+3x+2x=264.$

To clear an equation of fractions multiply each term of both members by the least common denominator of the fractions.

3. $\frac{x}{2}+\frac{x}{3}=35.$
4. $\frac{x}{3}+\frac{x}{4}=49.$
5. $\frac{x}{2}+\frac{2x}{3}=28.$
6. $\frac{3}{5}x+\frac{5}{7}x=92.$
7. $\frac{2x}{3}+\frac{3x}{4}=102.$
8. $2\frac{7}{8}x=115.$
9. $\frac{4x}{5}-\frac{2x}{3}=48.$

10. $x - \frac{x}{40} = 156.$

11. $3\frac{5}{8}x = 116.$

12. $\frac{3x}{2} = 27.$

13. $1\frac{1}{2}x = 27.$

14. $\frac{11x}{4} = 22.$

15. $2\frac{3}{4}x = 44.$

16. $2x + \frac{3x}{4} = 33.$

17. $3\frac{1}{2}x - 2\frac{3}{4}x = 45.$

18. $x + \frac{x}{5} = 24.$

19. $\frac{75x}{100} - \frac{33x}{50} = 81.$

20. $\frac{8x}{3} - \frac{2x}{5} = 136.$

21. $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 39.$

22. $x - \frac{x}{2} - \frac{x}{3} = 37.$

23. $\frac{4x}{5} - \frac{2x}{9} + \frac{3x}{4} = 239.$

24. $\frac{5x}{9} + \frac{2x}{3} - \frac{x}{2} = 52.$

25. $x - \frac{3x}{4} = 80.$

26. $x + 2x + \frac{3x}{7} = 24.$

530. Written Problems.

1. Divide 100 into two parts, one of which shall be $1\frac{1}{2}$ times the other.

2. After losing $\frac{1}{8}$ of his money, a man has \$714. How many dollars had he at first?

$$\left(x - \frac{x}{8} = 714\right).$$

3. A horse was sold for \$240, the seller thereby gaining one-third of what he originally paid for it. How much did he pay for it?

$$\left(x + \frac{x}{3}\right).$$

4. One-half of a number added to one-fourth of the same number equals $66\frac{3}{4}$. What is the number?

5. The difference between $\frac{3}{4}$ of a number and $\frac{3}{5}$ of the same number is 15. Find the number.

6. One number is $\frac{3}{8}$ of another. Their sum is 55. What are the numbers?

7. Find a fraction equivalent to $\frac{7}{8}$, the sum of its numerator and its denominator being 60.

(Let $7x$ = numerator and $8x$ = denominator.)

8. Find a fraction equivalent to $\frac{3}{7}$, the difference between its numerator and its denominator being 24.

9. The sum of two numbers is 480, and the quotient obtained by dividing the greater by the less is 7. What are the numbers?

10. Find two numbers whose difference is 522 and whose quotient is 30.

11. A boy buys apples at $2¢$, pears at $3¢$, and oranges at $4¢$, the same number of each. How many of each does he buy, if he pays $81¢$ for all?

12. A girl bought 70 cents' worth of peaches and plums. She paid $3¢$ each for the peaches and $2¢$ each for the plums, buying four times as many of the former as of the latter. How many of each did she buy?

13. $1500 is divided among three persons, the second of whom receives three times as much as the first, and the third three and one-half times as much as the first. Find the share of each.

14. A farmer paid for a cow three-sevenths as much as he paid for a horse. How much did he pay for each, if the latter cost $80 more than the former?

15. Three times a man's money increased by two-thirds of his money is equal to $1100. How much money has he?

16. After giving away $\frac{3}{8}$ of his marbles and losing $\frac{1}{4}$ of them, Joseph has 24 left. How many had he at first?

17. Bought a coat, a hat, and an umbrella for $15, paying for the hat $1\frac{1}{2}$ times as much as for the umbrella, and for the coat $3\frac{1}{3}$ times as much as for the hat. Find the price of each.

18. A merchant purchased two pieces of cloth for $240, paying for one piece twice as much per yard as for the other. The former contains 36 yards and the latter 48 yards. How much does he pay per yard for each?

19. A farmer sold 4 times as many cows as horses, receiving for all $840, at the rate of $40 for a cow and $120 for a horse. How many of each did he sell?

20. The weight of a team with a loaded wagon is 5500 pounds. The wagon weighs $\frac{2}{5}$ as much as the load. The team weighs twice as much as the wagon. How many pounds does the load weigh?

531. Oral Exercises.

Give values of x, y, z, etc.:

1. $x + 15 = 21$.
2. $2y + 15 = 21$.
3. $z - 7 = 21$.
4. $4w - 7 = 21$.
5. $\frac{v}{2} + 3 = 8$.
6. $\frac{x}{2} - 3 = 12$.
7. $3y + 6 = 15$.
8. $7y - 13 = 15$.
9. $9y + 13 = 58$.
10. $3y - 10 = 56$.
11. $\frac{3v}{4} + 1 = 7$.
12. $\frac{4w}{5} - 1 = 11$.

532. If $x + 15 = 21$, $x = 21 -$ what?

When $x - 7 = 21$, $x = 21 +$ what?

If in the equation $2x + 15 = 21$, we take away 15 from the first member, what must we do to the second member to preserve the equality?

If equals are subtracted from equals, the remainders are equal.

By *transposing* we mean bringing the unknown quantities (x, y, z, etc.) to one side of the equation and the known quantities to the other.

NOTE. — In bringing a quantity from one side of the equation to the other, the *sign* of the quantity is changed.

533. Written Exercises.

Find values of the unknown quantities.

NOTE. — Clear of fractions when necessary; then transpose.

1. $x + 37 = 56$.
2. $4x - 5 = 83$.
3. $3x - 43 = 98$.
4. $7x + 13 = 111$.
5. $x + 3x = 25 + 11$.
6. $5x = x + 40$.
7. $3x - 20 = x - 8$.
8. $12 - 3x = 45 - 4x$.
9. $3x - 6 = 48 + x$.
10. $3x + 6 = 9 - 2x + 12$.
11. $2x - 2 - 16 = x + 10$.
12. $\frac{x}{3} - 8 = 24$.
13. $\frac{x}{6} + 4 - 7 = 21$.
14. $\frac{x}{2} + \frac{x}{3} = 10 + 5$.
15. $7x - 5x = 20 + x + 4$
16. $6x - 14 = 16 + x$
17. $2x - 11 + 6x - 60 = 5x + 25$.
18. $\frac{x}{2} + \frac{x}{3} - 5 = 10$.
19. $2x - 6 = 16 + \frac{x}{2} - \frac{x}{3}$.
20. $2x + \frac{3x}{5} - \frac{x}{2} = \frac{3x}{4} + 27$.

534. Written Problems.

1. The sum of three numbers is 51. The second is 5 less than the first, and the third is 10 less than the first. What are the numbers?

Let x = first number,

$x - 5$ = second number,

$x - 10$ = third number;

$x + x - 5 + x - 10 = 51$.

Transposing, $x + x + x = 51 + 5 + 10$,

$3x = 66$,

$x = 22$, first number,

$x - 5 = 17$, second number,

$x - 10 = 12$, third number.

2. Add 45 to four times a number, and you will have seven times that number. What is the number?

$(7x = 45 + 4x.)$

3. Nine times a number less 27 equals six times the number. Find the number.

4. Two boys have together 48 marbles. One has 18 more than the other. How many has each?

$(x,\ x + 18.)$

5. The length of a rectangular lot is 75 feet more than the breadth. The distance around it is 250 feet. What are its dimensions?

6. A piece of land containing 86 acres is to be divided into two fields, one of which shall be 8 acres larger than the other. How many acres in each field?

7. At a certain election 2436 votes were cast for two candidates, the successful one receiving 318 more votes than his opponent. How many votes did each receive?

8. A man, being asked his age, replied that if he were half as old again and 7 years more he would be 100. What was his age?

9. The sum of two numbers is 96, and their difference is 72. Find the numbers.

(Let x = less, $x + 72$ = greater.)

10. After paying $\frac{1}{3}$ and $\frac{1}{4}$ of my debts, I still owe \$ 45. How much did I owe originally?

$$x - \frac{x}{3} - \frac{x}{4} = 45.$$

11. Divide 45 into two parts, one of which shall be 6 less than twice the other.

12. William has \$ 5 more than John, and three times William's money added to five times John's would be \$ 103. How many dollars has each?

13. I bought 3 cows and 4 horses for $635, paying $80 apiece less for the cows than for the horses. How many dollars apiece did I pay for each?

14. Mary has a dollar in dimes and five-cent pieces. She has 11 more of the latter than of the former. Find the number of pieces of each denomination.

15. Divide 100 into two parts whose difference shall be 48.

16. In a class of 54 pupils, the girls outnumber the boys by 12. How many are there of each?

17. $18,000 is divided among three persons, the second of whom receives $2400 more than the first, and the third of whom receives $2400 more than the second. Find the share of each.

18. The greater of two numbers is 11 more than 3 times the less. Their difference is 33. What are the numbers?

19. A boy spent a dollar for postal cards, 2-cent stamps, and 5-cent stamps. He bought 15 more 2-cent stamps than 5-cent stamps, and 15 more postal cards than 2-cent stamps. How many of each did he buy?

Let x = number of 5-cent stamps,

then $x + 15$ = number of 2-cent stamps,

$x + 30$ = number of postal cards.

$5x$ = value of 5-cent stamps,

$2x + 30$ = value of 2-cent stamps,

$x + 30$ = value of postal cards.

$$5x + 2x + 30 + x + 30 = 100$$

20. A farmer has 88 head of stock — horses, cows, and sheep. He has 17 more cows than horses, and the number of sheep is 22 greater than that of the cows and horses together. How many are there of each?

ADDITION OF ALGEBRAIC QUANTITIES.

535. Oral Exercises.

Add:

1.	2.	3.	4.	5.
2 fours	6 hundredths	\$4	$3¢$	$7x$
3 fours	8 hundredths	\$5	$5¢$	$4x$
4 fours	10 hundredths	\$7	$8¢$	$2x$
5 fours	12 hundredths	\$8	$9¢$	$5x$
? fours	? hundredths	\$?	$?¢$	$?x$

When no coefficient is expressed, 1 is understood. Thus, $abc = 1\,abc$.

Where no sign is expressed, + is understood.

6.	7.	8.	9.	10.
$-2a$	$+3x$	$-5xy$	$9abc$	$-24xyz$
$-4a$	$+4x$	$-4xy$	$15abc$	$-5xyz$
$-6a$	$+5x$	$-xy$	$6abc$	$-xyz$
$-7a$	$+10x$	$-2xy$	abc	$-15xyz$
$-19a$	$+?x$	$-?xy$	$?abc$	$-?xyz$

NEGATIVE QUANTITIES.

536. Suppose three men as follows:

The first man has \$5 and owes nothing.
The second man has \$5 and owes \$5.
The third man has nothing and owes \$5.
The first man is worth \$5.
The second man is worth nothing.
The third man is worth 5 less than nothing. So we may say he is worth $-\$5$.

537. Quantities like $-\$5$, -17, and $-2a$ are called negative quantities.

What sign precedes a negative quantity?

Quantities with a plus sign expressed or understood are called positive quantities.

538. Oral Exercises.

1. A man bought a horse for $100. He sold it for $110. What was his gain?

2. A man bought a horse for $200. He sold it for $175. What was his gain? *Ans.* $-\$25$.

3. A man earned $60 during November. He spent $45. How much did he save?

4. A man earned $60 during December. He spent $70. How much did he save?

5. A man went north from a starting point 10 miles. He then went south 8 miles. How far north of his starting point was he then?

6. A man went north from a starting point 10 miles. He then went south 14 miles. How far north of his starting point was he then?

7. What is meant by a gain of $-\$5$?

8. What is meant by -5 miles east?

9. What is the meaning of this statement? A woman is -3 inches taller than her husband.

10. John is 3 years older than Mary. Mary is how many years older than John?

ADDITION OF POSITIVE AND NEGATIVE QUANTITIES.

539. Preliminary Exercises.

1. On Monday an agent makes $6 above his expenses; on Tuesday, $-\$3$; on Wednesday, $4; on Thursday, $8; on Friday, $-\$2$; on Saturday, $7.

2. Another agent's profits for the days of the week are $-\$3$, $2, $-\$2$, $4, $-\$1$, $-\$2$.

We may put down the above statements thus:

$ 6	$ −3
−3	2
4	−2
8	4
−2	−1
7	−2
$ 20	$ −2

In the first example we add all the positive quantities, $ 6 + $ 4 + $ 8 + $ 7 = $ 25. Then we add all the negative quantities, − $ 3 − $ 2 = − $ 5. Adding $ 25 and − $ 5 the result is $ 20.

In the second example we add all the positive quantities, and get $ 6. The sum of the negative quantities is − $ 8. Adding $ 6 and − $ 8 the result is − $ 2.

Can you give the rule for addition where the quantities have different signs? Which sign does the sum take?

540. Written Exercises.

Add:

1.	2.	3.	4.	5.
$-2a$	$7x$	$-5xy$	$-9abc$	$-24xyz$
$-4a$	$-4x$	$-4xy$	$15abc$	$5xyz$
$-6a$	$-2x$	xy	$6abc$	xyz
$7a$	$5x$	$2xy$	$-abc$	$15xyz$

6. $3x+14$, $-7x+9$, -23, $4x-5$, $-2x$, and $3x+11$.

Write like quantities in the same column. Find the sum of the positive terms, also the sum of the negative terms; subtract the less from the greater, and prefix the sign of the greater.

$$\begin{array}{r} 3x+14 \\ -7x+\ 9 \\ -23 \\ 4x-\ 5 \\ -2x \\ \hline 3x+11 \end{array}$$

7. $4a+3x$, $-2a$, $-7x-3a$, $-5x$, $-9a+x$.

8. $-3b+c$, $4a+6b$, $5b-9c$, $-3a$, $-2a-3b+4c$

9. $\frac{1}{2}x-8$, $-x+4$, $-\frac{1}{2}x-3$, $7x+16$, $-5x-10$.

10. $4x+23$, $-8x+2\frac{1}{2}$, $-\frac{2}{3}x+11$, $-x+5$, $9x-3$.

SUBTRACTION OF ALGEBRAIC QUANTITIES.

541. Preliminary Exercises.

1. A man sold a horse for $100 at a gain of $25. Find the cost. (Cost = selling price − gain.)

	$100 = selling price	$100
subtract	25 = gain	or add −25
remainder	$75 = cost	$75

2. A man sold a horse for $100 at a gain of −$25. Find the cost.

	$100 = selling price	$100
subtract	−25 = gain	or add +25
	$125 = cost	$125

In the first of the above examples, subtracting +$25 is the same as adding −$25.

In the second of the above examples, subtracting −$25 is the same as adding +$25.

We changed the first example from subtraction to addition by changing the sign of the subtrahend from + to −.

We changed the second example from subtraction to addition by changing the sign of the subtrahend from − to +.

To subtract in algebra, change the sign of the subtrahend and proceed as in addition.

3. Add 7 and −3.
4. From 7 subtract −3.
5. From −7 subtract 3.
6. Add −7 and 3.
7. Add −7 and −3.
8. From −3 subtract −7.
9. Subtracting −7 is the same as adding what?
10. Is a positive quantity increased or decreased by subtracting a negative quantity?

NOTE. — When you become familiar with the process of subtraction it will not be necessary to *write* the subtrahend with a changed sign. You can *conceive* the sign changed and add.

542. Sight Exercises.

1. What is the difference between $+52°$ and $+33°$?

2. Between $+90°$ and $-10°$?
Show by a diagram.

3. A has \$600, B owes \$400. What are they worth together?
$(+\$600)+(-\$400)=?$

4. How much better off is A than B?
$(+\$600)-(-\$400)=?$

543. Written Exercises.

1. From $8a$ take $2a$.

$$\begin{array}{r} 8a \\ -\ 2a \\ \hline \textit{Ans.}\quad 6a \end{array}$$

2. From $2a$ take $8a$.

$$\begin{array}{r} 2a \\ -\ 8a \\ \hline \textit{Ans.}\quad -\ 6a \end{array}$$

3. From $-8a$ take $2a$.

$$\begin{array}{r} -\ 8a \\ -\ 2a \\ \hline \textit{Ans.}\quad -10a \end{array}$$

4. From $8a$ take $-2a$.

$$\begin{array}{r} 8a \\ +\ 2a \\ \hline \textit{Ans.}\quad 10a \end{array}$$

5. From $-8a$ take $-2a$.

6. From $-2a$ take $8a$.

7. From $-2a$ take $-8a$.

8. From $2a$ take $-8a$.

9. From $3x+14$ take $x+10$.

$$\begin{array}{r} 3x+14 \\ -\ x-10 \\ \hline \end{array}$$

10. From $5x-8$ take $-3x-9$.

11. From $x-28$ take $5x-37$.

12. From $7x+16$ take $9x-4$.

13. From $6x$ take $2x-5$.

14. From $8x$ take $9x+3$.

15. From $3x+2a-5$ take $x-a-9$.

16. From $7y-2z+b$ take $-8y+6b-z$.

17. From $c-d+e$ take $c+d-f$.

REMOVING PARENTHESES.

544. Written Exercises.

1. From $6x + 15y$ take $4x + 10y$.

We may write the above in a shorter way, thus:

$$6x + 15y - (4x + 10y).$$

The minus sign before the parenthesis shows that the quantity within the parenthesis is to be subtracted. What sign is before $10y$? What sign is understood within the parenthesis before $4x$? In subtraction, what is done with the signs of the subtrahend? If the whole expression is written without using the parenthesis, what must be done with the signs of the quantities within the parenthesis?

$a - (b - c)$ may be written $a - b + c$. Why?

$a + (b - c)$ may be written $a + b - c$. Why?

When removing a parenthesis preceded by a minus sign, change the signs of all quantities within the parenthesis.

545. Written Exercises.

Write the following without parentheses:

1. $57 + (33 - 16) = 74$.
2. $92 - (63 + 25) = 4$.
3. $(43 - 10) + (24 - 5) = 52$.
4. $(17 - 8) - (16 - 14) = 7$.
5. $75 + 4 \times (15 - 10) = 95$.
6. $75 - 4 \times (15 - 10) = 55$.
7. $4x + 5y + (2x - 6y) = 6x - y$.
8. $4x + 5y - (2x + 6y) = 2x - y$.
9. $4x - 5y - (x - 6y) = 3x + y$.
10. $4x - 5y - (-x + 6y) = 5x - 11y$.
11. $4x + 5y - (-2x - 6y) = ?$.
12. $-4x - 5y + (2x - 6y) = ?$.

546. Solve the following equations. Prove the correctness of your answers.

1. $6(2x-5)=5x+12.$

NOTE. $6(2x-5)$ means 6 times $(2x-5)$, or $12x-30$.

2. $7(x+2)=3x+50.$ 4. $3(16-x)=4(13-x).$

3. $5(3+x)+16=61.$ 5. $15(x-3)=2(189-16x)$

6. $38-(11-9x)=10x.$

Removing the parenthesis, we have

$$38-11+9x=10x.$$

Transposing, $$9x-10x=-38+11,$$

or, $$-x=-27.$$

Bringing $-x$ to the right side of the equation, and -27 to the left side, we have $$(+)27=(+)x.$$

In practice, however, when the result is such as the above, $-x=-27$, the signs of both members are changed, and the result becomes

$$x=27.$$

7. $2(x-1)-2(2x-19)=3(x-3).$

8. $6(2x-5)-5x=12.$

9. $5x-6(2x-5)=-12.$

10. $\dfrac{11-3x}{2}+5x=19.$

547. $\dfrac{18-6}{2}-\dfrac{24-4}{5}=2.$

Clear of fractions by multiplying both members of the equation by 10, and observe which sign must be changed to preserve the equality

When $x=6$, the above may be written

$$\frac{3x-6}{2}-\frac{4x-4}{5}=2.$$

Clearing of fractions, $15x-30-(8x-8)=20.$

Removing the parenthesis,

$$15x - 30 - 8x + 8 = 20.$$

Transposing, $15x - 8x = 20 + 30 - 8,$

or, $7x = 42,$

$$x = 6.$$

NOTE. — The horizontal line between the numerator and the denominator of the foregoing fractions has the effect of a parenthesis, the *entire* quantity above the line being divided by the number below.

Hence when an equation is cleared of fractions, what must be done with the signs of the terms obtained from a fraction with a minus sign?

$\frac{18-6}{2} = (18-6) \div 2,$ $\frac{24-4}{5} = \frac{1}{5}$ of $(24-4).$

$\frac{3x-6}{2} = \frac{1}{2}$ of $(3x-6),$ $\frac{4x-4}{5} = (4x-4) \div 5.$

548. Solve:

11. $\frac{x-1}{2} + \frac{x-2}{3} = 8.$

12. $\frac{x-1}{2} - \frac{x-2}{3} = 2.$

13. $\frac{x-1}{2} - \frac{x-2}{3} - \frac{x-3}{4} + 2 = 0.$

14. $\frac{2x-5}{2} + \frac{x-7}{4} = \frac{5x-3}{6}.$

15. $\frac{7x-8}{9} - (x+2) = \frac{4x+5}{6} - \frac{x+2}{3}.$

16. $\frac{40-5x}{3} = \frac{52+9x}{7}.$

17. $9\frac{3}{4} - \left(\frac{5}{4}x - \frac{x}{2}\right) = \frac{3}{8}x + 3\frac{3}{4}x.$

18. $2x = 3 + 2\frac{1}{4}x - (5 + \frac{2}{5}x) + 2\frac{3}{5}.$

19. $\frac{3}{2}x + 9 = 2x + (\frac{3}{5}x - \frac{1}{2}x).$

20. $\frac{x}{4} + \frac{x}{5} + \frac{x}{6} + \frac{x}{8} + 31 = x.$

21. $\frac{5}{4}x - 120 = \frac{x}{6} + 10.$

22. $x - 20 = \left(\frac{x}{7} + 15\right)4.$

23. $x + \frac{x}{3} + \frac{x}{4} = 19.$

24. $9(8x + 1) - 4 = 4(9x + 5) + 3.$

25. $2x + 3 = \frac{5x - 6}{2}.$

549. Written Problems.

1. A certain number is multiplied by $3\frac{2}{3}$; 7 is subtracted from the product; the remainder is divided by 16, giving a quotient of 3. What is the number?

2. Three-eighths of what number is 60 less than the number itself?

3. Four persons are of the same age. If the first were $\frac{1}{3}$ of his age older, the second $\frac{1}{4}$ of his age older, the third $\frac{1}{5}$ of his age older, and the fourth $\frac{1}{6}$ of his age older, the sum of their ages would be 99 years. What is the age of each?

4. A man spends $\frac{1}{2}$ of his earnings on board and lodging, $\frac{1}{8}$ on clothing and repairs, and $\frac{1}{5}$ on sundries. At the end of the year he has \$280 left. What are his yearly earnings?

$$x = \frac{x}{2} + \frac{x}{8} + \frac{x}{5} + 280.$$

5. A boy gave $\frac{1}{6}$ of his marbles to one companion, and $\frac{1}{8}$ of them to another. He then bought $\frac{1}{3}$ as many as he originally had, and had 4 marbles more than he had at first. How many did he have at first?

6. A father's age and a son's age added together amount to 138 years. Twelve years ago the father was twice as old as the son. How old is each now?

Let x = son's age 12 years ago. $2x$ = father's age then.

7. John has 80 cents, and William has 60 cents. How many cents should William give John so that the latter might have $2\frac{1}{2}$ times as much money as the former?

After William gives John x cents, the former has $(60 - x)$ cents, and the latter has $(80 + x)$ cents.

8. In how many years will a man, now 25, be double the age of his 11-year-old brother?

Let x = number of years. $25 + x$ and $11 + x$ = ages after x years.

9. A man has a cask of 60 gallons' capacity. He draws off one-fourth of its contents, and then fills it. If it takes 24 gallons to fill it, how many gallons did the cask originally contain?

10. A number is divided by 3, and 40 is subtracted from the quotient, leaving a remainder of 104. What is the number?

11. The difference between two numbers is 430. When the greater is divided by the less, the quotient is 4, and the remainder is 76. What are the numbers?

$$\text{Let } x = \text{less.} \quad \frac{\text{greater}}{\text{less}} = 4 + \frac{76}{\text{less}}.$$

12. A person pays \$103 with 29 \$2 and \$5 bills. How many are there of each denomination?

13. A father is 30 years older than his daughter. In 4 years, his age will be four times her age. What are their present ages?

x and $x + 30$ = present ages. $x + 4$ and $x + 34$ = ages 4 years later.

14. The product of two numbers is 180. If the smaller number be increased by 3, the product of the two numbers will be 225. What are the numbers?

$$\text{smaller} = x; \quad \frac{180}{x} = \text{greater}.$$

15. A man's wages are \$1 per day more than his son's. For 33 days' work, the father receives \$12 more than the son earns in 40 days. Find the wages of each.

TWO UNKNOWN QUANTITIES.

550. Preliminary Problems.

1. I paid a dollar for two 25¢ balls and five bats. How much did I pay apiece for the latter?

2. When three times one number is added to five times another, the sum is 84. If the second number is 12, what is the first number?

3. A girl paid 75¢ for $\frac{1}{2}$ pound of tea and $2\frac{1}{2}$ pounds of coffee. The coffee cost 20¢ per pound. What was the price of the tea per pound?

4. A man sold pigs at \$5 each and lambs at \$8 each, receiving \$42. He sold 4 lambs. How many pigs did he sell?

5. Four times a father's age added to twice his daughter's age amounts to 180 years. The girl is 10 years old. What is the father's age?

6. Eight peaches and seven pears cost 44¢. The peaches cost 2¢ each. What is the cost of a pear?

7. Two pieces of cloth and eleven pieces of silk contain 152 yards. There are 10 yards in each piece of cloth. How many yards in each piece of silk?

8. Two-thirds of a yard of linen and three-fourths of a yard of lace cost 40¢. The price of the lace is 32¢ a yard. Find the price of the linen.

9. Three and one-half times one number added to four and one-third times a second number equals 60. The second number is 9. What is the first number?

551. Written Exercises.

Find the value of the unknown quantity:

1. $8x + 7y = 44$. When $x = 2$, find the value of y.
2. $3y + 5z = 34$. Find the value of z; $y = 3$.
3. $2x + 11z = 152$. $x = 10$; $z = ?$.
4. $14x + 7y = 98$. $x = 3\frac{1}{2}$; $y = ?$.

5. $\frac{2}{3}x + \frac{3}{4}z = 40.$ $z = 32.$
6. $9x - 25y = 8.$ $x = 12.$
7. $3\frac{1}{2}y + 4\frac{1}{8}z = 60.$ $z = 9.$
8. $16x - 19z = 49.$ $z = 5.$
9. $7y - 3z = 18.$ $y = 6\frac{1}{7}.$
10. $32x + 50y = 2600.$ $y = 20.$

552. Written Problems.

1. The cost of 3 apples and 2 peaches is 7 cents. The cost of 2 apples and 2 peaches is 6 cents.

Subtracting the second lot of fruit from the first lot we have 1 apple.

Subtracting the price of the second lot from the price of the first lot we have 1 cent. 1 apple costs 1 cent.

If equals are subtracted from equals, the remainders are equal.

2. A boy gave 25¢ for 3 lemons and 8 oranges, another boy paid 17¢ for 3 lemons and 4 oranges. How much did the lemons cost apiece?

x = cost of lemons,	$3x + 8y = 25$	(1)
y = cost of oranges,	$3x + 4y = 17$	(2)
Subtracting (2) from (1),	$4y = 8$	
The oranges cost 2¢ each,	$y = 2.$	

How much apiece was paid for the lemons?

3. If 3 coats and 14 vests cost $78, and 2 coats and 14 vests, at the same rate, cost $66, how much does 1 coat cost? What is the price of a vest?

4. Given $4x + 7y = 53,$ (1)
$2x + 3y = 25,$ (2)
to find the value of y.

First multiply (2) by 2, making it $4x + 6x = 50$. Why?

5. What is the value of x in equation (1) in above example, when the value found for y is substituted therein? Substitute the same value for y in equation (2) and find the value of x.

553. Written Exercises.

Find the values of x and y in the following equations:

1. $x + y = 15$, $2x + 3y = 38$.

2. $2x + 2y = 30$, $x + 3y = 27$.

3. $2x + 3y = 18$, $4x + 3y = 24$.

4. $2x + 3y = 40$, $3x + 2y = 35$.

Multiply first equation by 3, $6x + 9y = 120$.
Multiply second equation by 2, $6x + 4y = 70$.

5. $7x + 5y = 82$, $2x + 3y = 36$.

6. $5x + 9y = 14$, $9x + 5y = 14$.

7. $3x + 5y = 17$, $8x + 2y = 17$.

8. $2x - 3y = 18$, $3x + 5y = 65$.

Given $\left.\begin{matrix}(1)\quad x + 3y = 46,\\ (2)\quad 7x - 4y = 22.\end{matrix}\right\}$ To find values of x and y.

Multiply (1) by 7, $7x + 21y = 322$

(2) $7x - 4y = 22$ Subtract.

$25y = 300$

$y = 12$

Substituting this value of y in (1), we have

$x + 36 = 46$,

$x = 46 - 36 = 10$. *Ans.* $x = 10$, $y = 12$.

9. $x + y = 18$, Add or subtract.
$x - y = 4$.

10. $4x + 3y = 17$, (1) Multiply (2) by 2 and subtract.
$2x - y = 1$. (2)

11. $3x + 4y = 48$, Add.
$x - 4y = 0$.

12. $3x + 5y = 13$, (1) Multiply (1) by 7 and (2) by 3.
$7x + 3y = 13$. (2) Subtract.

13. $4x+5y=32$, Add.
$6x-5y=-2$.

14. $3x+4y=3$, (1) Multiply (2) by 2. Add.
$12x-2y=3$. (2)

15. $5x=6y+5$, Transpose.
$3x=5y-4$.

16. $3x+5y+8=0$,
$2x-y-12=0$.

17. $y-2x=8x-1$,
$2y-4x=y+x+9$.

18. $5x+7y=55$, (1)
$9x+18y=126$. (2)
Divide (2) by 9 getting $x+2y=14$. (3)
Then multiply (3) by 5.

19. $\frac{x}{4}+\frac{2y}{3}=17$. Clear of fractions.
$\frac{5x}{4}+\frac{5y}{8}=20$.

20. $\frac{1}{2}x+\frac{1}{3}y=42$,
$\frac{1}{5}x+\frac{1}{7}y=17\frac{2}{5}$.

21. $23x-7y=3x+51$,
$11y=15x+2$.

22. $x+y=100{,}000$,
$\frac{5x}{100}+\frac{4y}{100}=4{,}640$.

23. $\frac{3x+7}{3y-4}=5$,
$\frac{7x-6}{5y+3}=2$.

24. $4\frac{1}{4}x+3\frac{2}{3}y=67$,
$7\frac{1}{2}x-5\frac{1}{3}y=12$.

25. $3(x+7)=9(y-9)$,
$4(3x-8)=17y-155$.

26. $2(x-11)-2(y-9)=6$,
$\frac{x+9}{y-3}=\frac{32}{15}$.

27. $\frac{x-4}{3}+\frac{y-1}{4}=5$,
$\frac{x-4}{3}-\frac{y-1}{4}=1$.

28. $\frac{2x+5y+3}{3x-4y-2}=6$,
$\frac{4x-7y+5}{x-2y+2}=5$.

554. Written Problems.

1. The sum of two numbers is 37. Twice the first added to three times the second is 96. What are the numbers?

Let x = first number; y = second number.

2. The difference between two numbers is 28. Five times the first less twice the second is 197. What are the numbers? $x - y = 28$; $5x - 2y = 197$.

3. The product of the first of two numbers by 5, added to the product of the second by 3, gives 37. The product of the first by 6, diminished by five times the second, equals 10. Find the numbers.

4. Divide 65 into two parts whose difference shall be 19.

Let x and y = parts. Solve also by one unknown quantity.

5. A person pays \$103 with 32 bills, some of them \$2 bills, the others \$5 bills. How many of each does he use?

6. For 25 head of pigs and sheep, a farmer received \$145. How many of each did he sell, if he sold the former at \$7 each, the latter at \$5 each?

7. 10 oranges and 4 peaches cost 38¢; 6 oranges and 7 peaches cost 32¢. Find the cost of an orange. Of a peach.

8. 5 pounds of tea and 3 pounds of coffee cost \$3.75; 8 pounds of tea and 1 pound of coffee cost \$5.05. What is each worth per pound?

9. A farmer buys a certain number of horses at \$125 each and a certain number of cows at \$40 each. They cost together \$740. If he had bought half as many horses and twice as many cows they would have cost \$730. How many of each did he buy?

10. A man paid 75¢ for 2 pounds of raisins and 3 pounds of cheese. 5 pounds of raisins and 2 pounds of cheese at the same prices would have cost 94¢. What did each cost per pound?

11. The sum of two numbers is 19 The sum of the second number and ten times the first, minus the sum of the first and ten times the second, equals 45. What are the numbers?

12. Reduce $\frac{5}{13}$ to an equivalent fraction, the sum of whose numerator and denominator shall be 126.

x = numerator; y = denominator.

$$\frac{x}{y} = \frac{5}{13};\ x + y = 126.$$

13. What fraction equivalent to $\frac{8}{15}$ has 147 for the difference between its numerator and denominator?

$$x - y = -147. \quad \text{Why?}$$

14. 10 pounds of coffee at 30¢ per pound are mixed with x pounds of coffee at 25¢ per pound. What is x equal to, when the mixture is worth 26¢ per pound?

$$25x + (10 \times 30) = 26\,(10 + x).$$

15. A grocer mixes green tea costing 60¢ per pound with black tea costing 40¢ per pound. He uses 100 pounds in all, and the mixed tea costs him 48¢ per pound. How many pounds of each does he use?

Let x = number of pounds of black tea; y = number of green. Then $x + y$ = number of pounds of mixed tea.

$$x + y = 100;\ 40x + 60y = 48\,(x + y).$$

THREE UNKNOWN QUANTITIES.

555. 1. Given the following:

$$3x + 2y - z = 12, \ (a)$$
$$5x - 4y + 3z = 16, \ (b)$$
$$2x + 3y + 2z = 35, \ (c)$$

to find the values of x, y, and z.

$$\begin{array}{lr} (a) \text{ multiplied by } 5, & 15x + 10y - 5z = 60 \\ (b) \text{ multiplied by } 3, & 15x - 12y + 9z = 48 \\ \text{Subtract,} & 22y - 14z = 12 \quad (d) \end{array}$$

an equation containing only two unknown quantities.

$$\begin{array}{lr} (b) \text{ multiplied by } 2, & 10x - 8y + 6z = 32 \\ (c) \text{ multiplied by } 5, & 10x + 15y + 10z = 175 \\ \text{Subtract,} & -23y - 4z = -143 \quad (e) \end{array}$$

an equation containing only two unknown quantities.

Compare the two equations (d) and (e), which contain the same two unknown quantities.

$$\begin{array}{lrcr} (d) \text{ multiplied by } 2, & 44y - 28z & = & 24 \\ (e) \text{ multiplied by } 7, & 161y - 28z & = & -1001 \\ \text{Subtract,} & 205y & = & 1025 \\ & y & = & 5 \end{array}$$

Substituting this value of y in (d), we have

$$110 - 14z = 12, \quad -14z = -98, \quad z = 7.$$

Substituting values of y and z in (a), we have

$$3x + 10 - 7 = 12, \quad 3x = 9, \quad x = 3.$$

$$\left.\begin{array}{l} \textit{Ans. } x = 3, \\ y = 5, \\ z = 7. \end{array}\right\}$$

2. Find the values of the unknown quantities in the following equations:

$$x - 3y + 2z = 3, \quad (a)$$
$$2x + y + 3z = 22, \quad (b)$$
$$5x + 2y + 7z = 51. \quad (c)$$

Multiply (a) by 2, and subtract from (b). Multiply (a) by 5, and subtract from (c). This gives two equations, each of which contains two unknown quantities.

Compare these two resulting equations, and eliminate y.

3. $5x-2y+z=10$, (a)
$3x+8y-5z=120$, (b)
$7x-3y-2z=8$. (c)

Eliminate z by comparing (a) and (b), multiplying the former by 5. Compare (a) and (c), multiplying the former by 2.

4. $13x-4y+15z=317$,
$7x+2y-3z=89$,
$21x-17y+9z=-104$.

5. $-8x+y-12z=-259$,
$7x-4y+25z=418$,
$13x+2y-41z=-500$.

6. $\frac{x}{3}+\frac{x+y}{3}=14$,

$\frac{x+y}{2}-\frac{x-y}{6}=16$.

7. $\frac{3x-5y}{2}+3=\frac{2x+y}{5}$,

$8-\frac{x-2y}{4}=\frac{x}{2}+\frac{y}{3}$.

8. $2+\frac{5x-6y}{13}=4y-3x$,

$12+\frac{5x-6y}{6}=2y+\frac{3x-2y}{4}$.

9. $\frac{5x-3}{4}-\frac{3x-19}{4}=2-\frac{3y-x}{6}$,

$\frac{2x+y}{2}-\frac{9x-7}{8}=\frac{3y+9}{4}-\frac{4x+5y}{16}$.

556. Written Problems.

1. A man placed $\frac{2}{3}$ of his capital at 5% and the other third at 6%. At the end of a year, capital and interest amounted to $31,600. What was his capital?

$$\frac{2x}{3} \times \frac{5}{100} \text{ and } \frac{x}{3} \times \frac{6}{100} = \text{interest.}$$

2. A has 18 chestnuts more than B. If each finds 4 more, A will have four times as many as B. How many chestnuts has each?

3. Two mechanics earn together $8 per day. One works 23 days and the other 17 days, for which they receive together $166. What does each earn per day?

4. The sum of the first and the second of three numbers is 55, of the first and the third 62, of the second and the third 83. What are the numbers?

SUGGESTION. — Add together the three equations.

5. The sum of two numbers is 53. Four times the first is 20 more than twice the second. Find the numbers.

6. A certain sum of money is divided among four persons. The first takes $\frac{1}{5}$ of it, the second takes $\frac{4}{9}$ of the remainder, the third takes $\frac{2}{5}$ of what then remains, the fourth receives the balance, $24. What is the share of each of the other three?

7. A merchant sold a lot of goods for $510, thereby losing $\frac{3}{20}$ of their cost. What did the goods cost?

8. A man collected a bill for a physician and deducted $\frac{1}{50}$ of the amount for his services. If he gave the physician $147, what was the amount collected?

9. Divide $130\frac{1}{4}$ acres of land among three persons, giving the first $27\frac{1}{2}$ acres more than the second, and the second $13\frac{3}{4}$ acres more than the third.

10. A merchant has sold $\frac{4}{7}$ of a piece of cloth, and has remaining 16 yards more than $\frac{1}{5}$ of the piece. How many yards did the piece contain originally?

11. A servant is engaged for a year for \$280 and a suit of clothes; he leaves at the end of six months, and receives \$130 and the suit. What is the value of the clothes?

Yearly wages $= 280 + x$. Wages for six months $= 140 + \frac{x}{2} = 130 + x$.

EXPONENTS.

557. (1) x^2 means x times x, or xx.
(2) y^3 means y times y times y, or yyy.
(3) a^4 means $aaaa$.

In (1) 2 is an exponent.
In (2) 3 is an exponent.
In (3) 4 is an exponent.

Notice that the exponent is written above the quantity to which it belongs.

On which side of the quantity is it written?

How many times is the quantity used as a factor if the exponent is 2? If the exponent is 3? If the exponent is 5?

Tell two facts concerning the location of the exponent. Tell one fact concerning the meaning of the exponent.

What is an exponent?

NOTE. — When no exponent is written, 1 is understood.

558. Oral Exercises.

If $a = 2$, and if $b = 3$, and if $c = 5$,

1. $a^2 = ?$
2. $a^2 + a = ?$
3. $a^2 + b + c = ?$
4. $a^2 + b^2 = ?$
5. $b^2 - a^2 = ?$
6. $a^2 + 2ab + b^2 = ?$
7. $a^3 + a^2 + a = ?$
8. $a^3 + b^2 + c = ?$
9. $a^4 + b^3 + c^2 = ?$
10. $a^2 + ? c = 19$.
11. $a^3 + b^3 = a^2 + b^2 + c^2 - ?$
12. $b^2 + a^4 = c^?$.

559. $3b^2$ means $3b$ times b and *not* $3b$ times $3b$. The exponent ^{2} belongs to the letter b and *not* to the expression $3b$.

ab^3 means $abbb$ and not $ab\,ab\,ab$.

In $5ab^3$ to what part of the expression does the exponent belong?

If we use a parenthesis, the effect is different. $(3b)^2$ means $3b$ times $3b$. $(ab)^3$ means $ab\,ab\,ab$.

What does $(2a)^4$ mean?

560. Oral Exercises.

If $a = 2$, and if $b = 3$, and if $c = 5$,

1. $5a^2 = ?$
2. $(5a)^2 = ?$
3. $(2b)^2 - 2b^2 = ?$
4. $4b^2 - c^2 = ?$
5. $(3a)^3 - 3a^3 = ?$
6. $abc^2 = ?$
7. $7ab^2 = ?$
8. $7a^2b = ?$
9. $abc^2 = ?$
10. $a^2b^2c = ?$
11. $c^2 - a^2 = ?\,(c - a)$.
12. $4b^2 - 2ac = a^2$.

MULTIPLICATION.

561. What coefficient is understood when none is written? What exponent is understood when none is written?

$2a$ times $3 = 6a$.

$2a$ times $3a = 6a^2$.

$3a$ times $4a = 12a^2$.

$5a^2$ times $2a = 10a^3$.

$5a$ times $2b = 10ab$.

Multiply together the numerical coefficients and affix all the letters, giving each letter the sum of the exponents of that letter in both factors.

562. Oral Exercises.

Multiply:

1. $3a$ by $5a$.
2. $2ab$ by $3a$.
3. $2ab$ by $3b$.
4. $2a^2b$ by $3a$.
5. $2ab^2$ by $3a$.
6. $2a^2b^2$ by $3b$.
7. $5a^2b$ by $3c$.
8. $5ab$ by $3ac$.
9. $5ab$ by $3ab^2c^3$.
10. ab^2c^3 by abc.
11. $\frac{1}{2}ab$ by $6a^2$.
12. $10a^2b$ by $\frac{1}{5}a$.

SIGNS IN MULTIPLICATION.

563. Preliminary Exercises.

1. $+a$ times $+b = +ab$.

Here the two factors, $+a$ and $+b$, have like signs (both are $+$). The product has the $+$ sign.

2. $-a$ times $-b = +ab$.

Here the two factors have like signs (both are $-$). The product has the $+$ sign.

3. $-a$ times $+b = -ab$.

Here the two factors have unlike signs (one $+$ and one $-$). The product has the $-$ sign.

4. $+a$ times $-b = -ab$.

Here the two factors have unlike signs. The product has the $-$ sign.

From the above we may see the law for signs in multiplication.

Like signs give $+$.
Unlike signs give $-$.

Multiply:

5. $-3a$ by $5a$.
6. $2a$ by $4a$.
7. $-2a$ by $-4a$.
8. $2ab^2$ by $-3a$.
9. $-5a^2b$ by $-3c$.
10. $-5a^2b$ by $3c$.
11. $5a^2b$ by $-3c$.
12. $-5a^2b$ by -1.

564. Written Exercises.

Multiply:

1. a^2+3a by $2a$. *Ans.* $2a^3+6a^2$.
2. $4a-b$ by $2ab$. *Ans.* $8a^2b-2ab^2$.
3. $5a+3b$ by $4a$.
4. $ax-cx$ by $-3a$.
5. a^2+a+1 by a.
6. a^2+2a+1 by $-2a$.
7. a^2+a+1 by ab.
8. a^2-b^2 by a.
9. a^2-b^2 by $-a$.
10. ab^2c^3-a by ab.
11. $a^2+2ab+b^2$ by $2ab$.
12. $a^2-2ab+b^2$ by $-2ab$.

565. Written Exercises.

1. Multiply $x+2$ by $x+3$.

$$\begin{array}{ll} & x+2 \\ & x+3 \\ \hline \text{Multiplying } x+2 \text{ by } x, & x^2+2x \\ \text{Multiplying } x+2 \text{ by } 3, & \quad 3x+6 \\ \hline \text{Adding the two parts of the product,} & x^2+5x+6 \end{array}$$

Multiply each term of the multiplicand by each term of the multiplier and combine the products.

2. Multiply $x+3$ by $x-4$.

$$\begin{array}{r} x+3 \\ x-4 \\ \hline x^2+3x \\ \quad -4x-12 \\ \hline x^2-x-12 \end{array}$$

Multiply:

3. $x+3$ by $x+4$.
4. $x+5$ by $x-2$.
5. $x+8$ by $x-9$.
6. $2x+5$ by $x+2$.
7. $2x-8$ by $x+9$.
8. $3x+1$ by $x+7$.
9. $2x+1$ by $2x+1$.
10. $x-5$ by $x+4$.

566. Written Exercises.

Find products:

NOTE: $(x-3)(x+9)$ means $x-3$ multiplied by $x+9$.

1. $(x-3)(x+9)$.
2. $(x-6)(x+7)$.
3. $(x-5)(x+5)$.
4. $(x+5)(x-5)$.
5. $(2x-6)(x+1)$.
6. $(x-6)(2x+1)$.
7. $(2x-6)(3x+3)$.
8. $(3x+6)(2x-3)$.
9. $(2x+3)(2x-3)$.
10. $(x-5)(x-4)$.
11. $(x-7)(x-9)$.
12. $(x-7)(x-7)$.
13. $(x-5)(x-9)$.
14. $(x+5)(x+5)$.
15. $(x-3)(x+8)$.
16. $(x+7)(x-6)$.
17. $(x-4)(x-7)$.
18. $(2x-4)(3x-6)$.
19. $(2x+6)(3x-7)$.
20. $(2x+7)(3x+3)$.
21. $(2x-3)(3x-2)$.
22. $(2x-3)(2x+3)$.
23. $(2x+9)(4x-6)$.
24. $(3x-4)(3x+4)$.

567. Written Exercises.

1. Multiply $x+y$ by $x+y$.

$$\begin{array}{r} x+y \\ x+y \\ \hline x^2+xy \\ \quad xy+y^2 \\ \hline x^2+2xy+y^2 \end{array}$$

Multiply:

2. $a+b$ by $a+b$.
3. $a-y$ by $a-y$.
4. $a+3x$ by $a+3x$.
5. $2a+x$ by $a+2x$.
6. $x+y$ by $x-y$. *Ans.* x^2-y^2.
7. $a+1$ by $a-1$.
8. $m+n$ by $m-n$.
9. a^2+b by a^2+b.
10. a^2+b^2 by $a-b$.
11. a^2+a+1 by $a-1$. *Ans.* a^3-1.
12. a^2-a+1 by $a+1$.
13. x^2+x+1 by $x-1$.

TERMS.

568. Preliminary Exercises.

1. $2abc$.
2. $2+a+b+c$.
3. $a^2+2ab+b^2$.
4. $7ab^2-1$.

The first of the above expressions has one term.
The second has 4 terms.
The third has 3 terms.
The fourth has 2 terms.

An expression containing one term is called a monomial; one containing two terms, a binomial; one containing three terms, a trinomial; one containing four or more, a polynomial.

How many terms has each of the following?

5. $2acx^2y$.
6. $a+1$.
7. a^2+a+11.
8. $11a^2+a$.
9. $a^2+a+abc$.
10. $a^2+a+b+c$.

LIKE TERMS.

569. Like terms are those containing the same letters and the same exponents for each letter.

570. Oral Exercises.

Which of the following expressions contain like terms?

1. a and b.
2. $2a$ and a.
3. a^2 and a.
4. a^2 and $2a$.
5. $2a^2$ and $3a^2$.
6. $2a^2$ and $2a^3$.
7. $2a^2b$ and $3a^2b$.
8. $2a^3b$ and $3a^2b$.
9. xy^2 and x^2y.
10. $5ax^2y$ and ax^2y.

COMBINING LIKE TERMS.

571. **Two or more like terms may be combined into a single term. Unlike terms cannot be so combined.**

572. Written Exercises.

Combine when possible:

1. $3\,abc^2 - abc + abc^2 = 4\,abc^2 - abc.$

The first and third terms are combined.

2. $3\,abc^2 - ab^2c + a^2bc.$

3. $3\,abc - 2\,abc + abc.$

4. $2\,abx - ab + 3\,bx.$

5. $a^3 - a^2 + 2\,a - 4 + 3\,a^2 - a.$

6. $4\,a^2 - 2\,ab + b^2 + a^2 - b^2.$

7. $8\,x^2y - xy + x - xy - 3\,x.$

8. $4\,m^2 + 4\,mn + n^2 - mn^2.$

573. Written Exercises.

1. $(a^2 + b)(a + b) = ?$

Multiplying by a, we have $a^3 + ab$. Multiplying a^2 by b, gives a^2b. As there is no like term, this product is placed after ab. The product of b by b is placed last.

$$\begin{array}{r} a^2 + b \\ a + b \\ \hline a^3 + ab + a^2b + b^2 \end{array}$$

Rearrange the terms in the order of the size of the exponents of a.

Ans. $a^3 + a^2b + ab + b^2.$

2. $(2\,ab - b)(b - 1) = ?$

3. $(2\,ab - b)(2\,a - 1) = ?$

4. $(2\,ab - b)(ab - b) = ?$

5. $(a^2 - b^2)(a + b) = ?$

6. $(a + b)(c + d) = ?$

7. $(3\,a + b)(a - ab) = ?$

8. $(2\,pq - x)(2\,pq + x) = ?$

9. $(x^2 + 4\,y^2)(4\,x - y) = ?$

10. $(x^2 + x - 2)(x - 2) = ?$

DIVISION.

574. Preliminary Exercises.

1. $6a \div 3 = 2a$.
2. $a^2 \div a = a$.
3. $6a^2 \div 3a = 2a$.
4. $5a^2x \div 5a^2 = x$.
5. $10a^2x \div 5x = 2a^2$.
6. $10a^2x \div 2ax = 5a$.
7. $(12a^3xy^2 - 27a^2x^2y) \div 3a^2xy = 4ay - 9x$.

In the above examples what is done with the coefficients? What is done with the exponents of the same letter?

When the divisor is a monomial, divide the numerical coefficient of each term of the dividend by the numerical coefficient of the divisor. Then write the letters of the dividend, giving each an exponent equal to the exponent in the dividend diminished by that in the divisor.

575. Like signs give $+$. Unlike signs give $-$.

576. Written Exercises.

Divide:

1. $5a^3$ by a^2.
2. $-x^3y^3$ by xy^2.
3. x^3y^3 by $-xy^2$.
4. $-x^3y^3$ by $-xy^2$.
5. $x^2 + 2xy$ by x.
6. $x^3 - 5x^2 + 3x$ by x.
7. $15x^4 - 10x^3 + 20x^2$ by $-5x^2$.
8. $4x^3y^3 - 3x^2y + xy^2$ by xy.
9. $-12x^3y^2 + 33x^2y^3 - 24xy^4$ by $-3xy^2$.
10. $-7x^5$ by $-7x^4$.
11. $7x^5$ by $-7x^5$.
12. $a^2 + a$ by $\frac{1}{2}a$. *Ans.* $2a + 2$.
13. $x^3 + 3x^2$ by $-\frac{1}{3}x$.
14. $6a^3 - 3a^2$ by $-9a^2$.
15. $-x + xy - xz$ by $-x$.
16. $\frac{1}{2}x^3y - 2x^2y^2$ by $-\frac{1}{2}x^2y$.

577. Written Exercises.

1. Divide $x^2 + 7x + 12$ by $x + 3$.

We write a division like the above as follows:

$$x^2 + 7x + 12 \mid \underline{x + 3}$$

The first term in the divisor is x.

The first term in the dividend is x^2.

Dividing x^2 by x we get x for the first term of the quotient, which we put down thus,

$$x^2 + 7x + 12 \mid \begin{matrix} \underline{x + 3} \\ x \end{matrix}$$

then we multiply the divisor by the first term of the quotient and write the result under the dividend.

$$\begin{matrix} & x^2 + 7x + 12 \mid \underline{x + 3} \\ (x + 3)\ x \text{ times} & x^2 + 3x \mid x \end{matrix}$$

Next, we subtract $x^2 + 3x$ from the dividend,

$$\begin{matrix} & x^2 + 7x + 12 \mid \underline{x + 3} \\ \text{subtract} & \underline{x^2 + 3x} \mid x \\ \text{remainder} & 4x + 12 \end{matrix}$$

Next, we divide the first term of the remainder by the first term of the divisor and get $+ 4$, which we write in the quotient thus,

$$\begin{matrix} x^2 + 7x + 12 \mid \underline{x + 3} \\ \underline{x^2 + 3x} \mid x + 4 \\ 4x + 12 \end{matrix}$$

Next, we multiply the divisor by 4 and write the product thus,

$$\begin{matrix} x^2 + 7x + 12 \mid \underline{x + 3} \\ \underline{x^2 + 3x} \mid x + 4 \\ 4x + 12 \\ (x + 3)\ 4 \text{ times} \quad 4x + 12 \end{matrix}$$

Subtracting, there is no remainder. The whole quotient is $x + 4$.

$$\begin{matrix} x^2 + 7x + 12 \mid \underline{x + 3} \\ \underline{x^2 + 3x} \mid x + 4 \quad \textit{Ans.} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{matrix}$$

2. Divide $x^2+18x+56$ by $x+4$.

$$\begin{array}{r|l} x^2+18x+56 & x+4 \\ \underline{x^2+\ 4x\ \ \ \ } & \underline{x+14} \quad \textit{Ans.} \\ 14x+56 & \\ \underline{14x+56} & \\ 0 & \end{array}$$

3. Divide $a^2-2ab-24b^2$ by $a+4b$.

$$\begin{array}{r|l} a^2-2ab-24b^2 & a+4b \\ \underline{a^2+4ab\ \ \ \ \ \ } & \underline{a-6b} \quad \textit{Ans.} \\ -6ab-24b^2 & \\ \underline{-6ab-24b^2} & \\ 0 & \end{array}$$

Prove that the answers in the above examples are correct.

578. Written Exercises.

Divide:

1. x^2+5x+6 by $x+3$.
2. x^3+5x^2+6x by $x+3$.
3. x^3+5x^2+6x by x^2+3x. *Ans.* $x+2$.
4. $x^2+7xy+10y^2$ by $x+2y$.
5. $x^2-7xy+10y^2$ by $x-2y$.
6. $3x^2+14xy+8y^2$ by $x+4y$.
7. $3x^2+10xy-8y^2$ by $x+4y$.
8. $3x^2-10xy-8y^2$ by $x-4y$.
9. $3x^2-14xy+8y^2$ by $x-4y$.
10. $3x^2+10xy-8y^2$ by $3x-2y$.
11. $8x^2+22xy+15y^2$ by $2x+3y$.
12. $8x^2-2xy-15y^2$ by $4x+5y$.
13. $8x^2-22xy+15y^2$ by $4x-5y$.
14. $n^2x^2+3anx+2a^2$ by $nx+a$. *Ans.* $nx+2a$.
15. $n^2x^2+anx-2a^2$ by $nx-a$.
16. $6a^2b^2-13abw+6w^2$ by $3ab-2w$.
17. $4x^2+2xyz-132y^2z^2$ by $2x-11yz$.

18. $4\,ax^2 + 2\,axyz - 132\,ay^2z^2$ by $2\,x - 11\,yz$.

19. $8\,ax^2 - 26\,axyz + 15\,ay^2z^2$ by $4\,ax - 3\,ayz$.

579. Written Exercises.

1. Divide $x^3 - 1$ by $x - 1$.

$$\begin{array}{l|l} x^3 - 1 & x - 1 \\ \underline{x^3 - x^2} & x^2 + x + 1 \quad \textit{Ans.} \\ x^2 - 1 & \\ \underline{x^2 - x} & \\ x - 1 & \\ \underline{x - 1} & \\ 0 & \end{array}$$

2. Divide $x^3 - 13\,x - 10 - 2\,x^2$ by $x - 5$.

The terms of the dividend should be arranged according to the size of the exponents of x, thus, $x^3 - 2\,x^2 - 13\,x - 10$.

$$\begin{array}{l|l} x^3 - 2\,x^2 - 13\,x - 9 & x - 5 \\ \underline{x^3 - 5\,x^2} & x^2 + 3\,x + 2 + \dfrac{1}{x-5} \\ 3\,x^2 - 13\,x - 9 & \\ \underline{3\,x^2 - 15\,x} & \\ 2\,x - 9 & \\ \underline{2\,x - 10} & \\ +\ 1 & \end{array}$$

Divide:

3. $a^3 - b^3$ by $a - b$.

4. $c^3 - 1$ by $c - 1$.

5. $c^3 + 1$ by $c + 1$.

6. $9\,a^4 - 16\,b^2$ by $3\,a^2 + 4\,b$.

7. $c^3 + 2$ by $c + 1$. *Ans.* $c^2 + c + 1 + \dfrac{1}{c+1}$.

8. $x^3 + 6\,x^2y + 12\,xy^2 + 8\,y^3$ by $x + 2\,y$.

9. $a^3 + 3\,a - 3\,a^2 - 1$ by $a - 1$. Rearrange dividend.

10. $x^3 + 27\,y^3$ by $x + 3\,y$.

11. $a^2 + 2\,ab + b^2 - 1$ by $a + b + 1$. *Ans.* $a + b - 1$.

12. $12\,a^3 - 20\,a^2 + 33\,a - 5$ by $6\,a - 1$.

13. $8\,ab^3 - 125\,ax^3y^3$ by $2\,ab - 5\,axy$.

14. $1 + 5\,a^3 - 6\,a^4$ by $1 - a + 3\,a^2$.

15. $6\,x^5 + x - 12\,x^2 + 9\,x^3 - 3 - 11\,x^4$ by $2\,x^3 - 3\,x^2 - 1$.

FACTORING.

580. The expression $x + xy$ may be divided by x; the quotient is $1 + y$. The factors of $x + xy$ are x and $1 + y$; that is, $(x + xy) = x(1 + y)$.

In a similar manner we find that the factors of $2a^2b + 4ab^2$ are $2ab$ and $a + 2b$; that is, $2a^2b + 4ab^2 = 2ab(a + 2b)$.

581. Factor:

1. $m + mn$.
2. $m^2n + 2m$.
3. $6x^2y - 3xy^2$.
4. $a^3 + a^4$.
5. $6b^3 - 9b^2$.
6. $a^3b + 6ab^3$.
7. $c^3 + c^2 + c$.
8. $7a^2b + 21a^2c$.
9. $4x^4 + 6x^3$.
10. $4x^4 + 6x^3 + 8x^2$.
11. $4x^4 + 6x^3 + 8x^2 + 10x$.
12. $4x^4 + 6x^3 + 8x^2 + 10x + 12$.
13. $12ab^2 - 9ac^2 + 6ac^3$.
14. $3ab^2 + 2a^2b + 2a^3$.
15. $3a^2b + 6ab^2 - 15ab^3$.
16. $x^3y^2z + xyz^2$.
17. $9m^2n^3x - 27m^3n^2y$.
18. $20ax^2 - 15bx^3 + 16cx^4$.
19. $20ax^2 + 15a^2x^3 - 20a^3x^4$.
20. $a^5x^3 + a^4x^4$.
21. $6a^2 + 4a^4 + 2a^6$.
22. $a^2x^4 + a^3x^3 + a^4x^4$.
23. $12m^3n + 5m^2n^2 + 15ny$.
24. $a^7 - a^5b^2 + a^4c^3$.
25. $70x^7 + 60x^6 - 50x^5$.

582. The square of $x + y$ is $x^2 + 2xy + y^2$.

Note that x^2 and y^2 are the squares of x and y, respectively, and that $2xy$ is twice the product of x and y.

The square of the sum of two quantities is equal to the square of the first, plus twice the product of the first and the second, plus the square of the second.

Any expression in the form of $x^2 + 2xy + y^2$ is composed of two equal factors.

$$a^2 + 2ab + b^2 = (a + b)(a + b) \text{ or } (a + b)^2.$$

583. Factor:

1. $c^2 + 2\,cd + d^2$.
2. $m^2 + 2\,m + 1$.
3. $4 + 4\,w + w^2$.
4. $a^2x^2 + 2\,axy + y^2$.
5. $r^2 + 2\,rst + s^2t^2$.
6. $e^2 + 6\,e + 9$.
7. $x^2 + 8\,x + 16$.
8. $4\,b^2 + 4\,bd + d^2$.
9. $a^2 + 4\,ay + 4\,y^2$.
10. $a^2 + 4\,ayz + 4\,y^2z^2$.
11. $4\,a^2x^2 + 4\,abx + b^2$.
12. $u^2 + 6\,uv + 9\,v^2$.
13. $a^4 + 2\,a^2b^2 + b^4$.
14. $c^2d^2 + 6\,cdm + 9\,m^2$.
15. $4\,a^2 + 12\,a + 9$.
16. $9\,a^2 + 12\,ab + 4\,b^2$.
17. $4\,a^2 + 4\,ac + c^2$.
18. $a^4 + 2\,a^2b + b^2$.
19. $a^2 + 2\,abx + b^2x^2$.
20. $a^2b^2 + 2\,abcd + c^2d^2$.
21. $m^2n^2 + 10\,mn + 25$.
22. $9\,b^2 + 30\,bc + 25\,c^2$.
23. $16 + 16\,xy + 4\,x^2y^2$.
24. $x^4 + 2\,x^2y^2z + y^4z^2$.
25. $a^2b^2 + 12\,abc + 36\,c^2$.
26. $4\,m^2 + 8\,mn + 4\,n^2$.

584. The square of $a - b$ is $a^2 - 2\,ab + b^2$.

Compare this form with that of Article 582, and note the difference in signs.

Give a general statement for the square of the difference of two quantities.

585. Factor:

1. $a^2 - 2\,ay + y^2$.
2. $1 - 2\,x + x^2$.
3. $m^2 - 2\,mnr + n^2r^2$.
4. $a^2 - 4\,a + 4$.
5. $9 - 6\,b + b^2$.
6. $r^2s^2 - 2\,rst + t^2$.
7. $c^2 - 4\,cd + 4\,d^2$.
8. $16\,x^2 - 8\,x + 1$.
9. $4\,b^2 - 4\,bc + c^2$.
10. $a^2y^2 - 4\,ayz + 4\,z^2$.
11. $4\,a^2y^2 - 4\,ayz + z^2$.
12. $9\,a^2 - 6\,ah + h^2$.
13. $b^2c^2 - 6\,bcd + 9\,d^2$.
14. $x^2 - 2\,x + 1$.
15. $4\,x^4 - 4\,x^2 + 1$.
16. $4\,x^2 - 12\,xy + 9\,y^2$.
17. $9\,y^2 - 12\,xy + 4\,x^2$.
18. $a^4 - 2\,a^2x + x^2$.
19. $9 - 12\,b + 4\,b^2$.
20. $4\,x^2 + 4\,x + 1$.
21. $x^2 + 4\,y^2 - 4\,xy$.
22. $16\,x^2 - 40\,xz + 25\,z^2$.
23. $b^4 - 2\,b^2cd^2 + c^2d^4$.
24. $25\,x^4 - 30\,x^2 + 9$.
25. $25\,x^5 - 30\,x^3 + 9\,x$.
26. $50\,x^5 - 60\,x^3 + 18\,x$.

586. The product of $a + b$ and $a - b$ is $a^2 - b^2$.

Give a general statement for the product of the sum and the difference of two quantities.

An expression consists of the difference of the squares of two quantities. What are the factors of the expression?

In factoring an expression first examine it to see if it contains a monomial factor. $b^2 - bc^2 = b(b^2 - c^2)$. The factors of $b^2 - c^2$ are $b + c$ and $b - c$. $b^3 - bc^2 = b(b + c)(b - c)$.

587. Factor:

1. $m^2 - n^2$.
2. $1 - m^2$.
3. $x^2 - 4$.
4. $c^2 - d^2y^2$.
5. $9 - v^2$.
6. $h^2 - 16$.
7. $4x^2 - y^2$.
8. $a^2x^2 - y^2$.
9. $x^2 - 4y^2$.
10. $4a^2b^2 - w^2$.
11. $aw^2 - 4a$.
12. $m^2 - 9n^2$.
13. $b^3 - 9b$.
14. $9m^2 - n^2p^2$.
15. $9b^2 - 4$.
16. $4c^2 - 9d^2$.
17. $bc^2 - 4bd^2$.
18. $a^4 - b^2$.
19. $x^6 - y^2$.
20. $xy^2 - a^2b^2x$.
21. $x^2y^2 - a^2b^2$.
22. $-m^2 + a^2x^2$.

588. The product of $x + 2$ and $x + 3$ is $x^2 + 5x + 6$.

Note that the coefficient of the second term of the product is the sum of the second terms of the factors; $5 = 2 + 3$.

Note that the last term of the product is the product of the second terms of the factors; $6 = 2$ times 3.

Factor $x^2 + 7x + 12$.

We must find two numbers whose sum is 7 and whose product is 12. These numbers are 3 and 4.

$$x^2 + 7x + 12 = (x + 3)(x + 4).$$

In a similar manner, we find

$$x^2 + 6x + 5 = (x + 1)(x + 5).$$

589. Factor:

1. x^2+4x+3.
2. m^2+6m+8.
3. $b^2+7b+10$.
4. c^2+7c+6.
5. a^2+8a+7.
6. $a^2+9a+14$.
7. $y^2+8y+12$.
8. $d^2+10d+16$.
9. $h^2+8h+15$.
10. $x^2+9x+18$.
11. $x^2+9x+20$.
12. $z^2+10z+21$.
13. $w^2+12w+20$.
14. $r^2+10r+9$.
15. $s^2+11s+18$.
16. $24+10a+a^2$.
17. $v^2+11v+24$.
18. $20+12x+x^2$.
19. $x^2+12x+32$.
20. $27+12x+x^2$.
21. $a^2+14a+24$.
22. $c^2+20c+19$.
23. $y^2+12y+35$.
24. $m^2+13m+30$.
25. $x^2+11x+30$.
26. $f^2+10f+9$.

590. The product of $x-2$ and $x-3$ is x^2-5x+6. Compare with Article 588 and note the difference in signs.

$$a^2-11a+10=(a-10)(a-1).$$

591. Factor:

1. $c^2-7c+12$.
2. d^2-5d+4.
3. $x^2-13x+22$.
4. $y^2-14y+33$.
5. $z^2-12z+11$.
6. $a^2-13a+40$.
7. $m^2-18m+32$.
8. $n^2-14n+13$.
9. $n^2-14n+45$.
10. $n^2-18n+45$.
11. $x^2-16x+28$.
12. $36-15z+z^2$.
13. $k^2-17k+30$.
14. $30-31b+b^2$.
15. $s^2-16s+55$.
16. $y^2-16y+63$.

17. $48 - 14\,c + c^2$.
18. $z^2 - 27\,z + 50$.
19. $w^2 + 20\,w + 51$.
20. $50 + 15\,t + t^2$.
21. $h^2 + 17\,h + 42$.
22. $80 + 18\,y + y^2$.
23. $d^2 + 17\,d + 72$.
24. $x^2 - 20\,x + 99$.
25. $99 - 100\,a + a^2$.

592. 1. The product of $x + 5$ and $x - 3$ is $x^2 + 2\,x - 15$.

2. The product of $x - 5$ and $x + 3$ is $x^2 - 2\,x - 15$.

What gives a minus sign in a product?

Why is the sign of the last term of the product minus in each of the above statements?

Note that in each of the products the coefficient of the second term is the *difference* between 5 and 3.

In the factors in the first statement, has the larger number a plus or a minus sign? What is the sign of the second term of the product?

In the factors in the second statement, has the larger number a plus or a minus sign? What is the sign of the second term of the product?

Factor $x^2 - 4\,x - 45$.

We must find two numbers whose product is 45 and whose difference is 4. We see that 9 and 5 are such numbers.

The sign of the second term in the given expression is minus, so we must give the minus sign to the larger of the two numbers which we have found.

$$x^2 - 4\,x - 45 = (x - 9)(x + 5).$$

In a similar manner, we would get

$$x^2 + 4\,x - 45 = (x + 9)(x - 5).$$

$$x^2 + x - 56 = (x + 8)(x - 7).$$

$$x^2 - x - 56 = (x - 8)(x + 7).$$

593. Factor:

1. $x^2 - x - 12$.
2. $x^2 + x - 12$.
3. $a^2 + 3a - 10$.
4. $a^2 - 3a - 10$.
5. $y^2 - 4y - 12$.
6. $y^2 + 4y - 12$.
7. $c^2 + 5c - 6$.
8. $d^2 - 7d - 18$.
9. $d^2 + 7d - 18$.
10. $k^2 - 7k - 8$.
11. $g^2 - 2g - 24$.
12. $x^2 - 5x - 24$.
13. $x^2 + 5x - 36$.
14. $x^2 - 5x - 36$.
15. $a^2 - 5a - 14$.
16. $b^2 - 7b - 30$.
17. $z^2 - 2z - 35$.
18. $m^2 + 4m - 60$.
19. $k^2 + 3k - 54$.
20. $x^2 + 2x - 48$.
21. $t^2 - 10t - 11$.
22. $y^2 + 7y - 60$.
23. $h^2 - 9h - 36$.
24. $a^2 - 2a - 120$.
25. $d^2 + 5d - 150$.
26. $d^2 - 5d - 150$.

594. If we divide $x^3 - y^3$ by $x - y$, the quotient is $x^2 + xy + y^2$.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

The difference of the cubes of two quantities may be divided by the difference of the quantities. The quotient consists of three terms, namely: *the square of the first, plus the product of the first and second, plus the square of the second.*

$$(a^3 + b^3) \div (a + b) = a^2 - ab + b^2.$$

Give a general statement for the quotient obtained by dividing the sum of the cubes of two quantities by the sum of the quantities.

Factor $a^3 + 8$.

$$8 = 2^3,$$

hence

$$a^3 + 8 = a^3 + 2^3,$$

$$a^3 + 2^3 = (a + 2)(a^2 - 2a + 2^2),$$

since

$$2^2 = 4,$$

$$a^3 + 8 = (a + 2)(a^2 - 2a + 4).$$

595. Factor:

1. $m^3 - x^3$.
2. $m^3 + x^3$.
3. $a^3 + 1$.
4. $a^3 - 1$.
5. $1 - a^3$.
6. $b^3 - c^3d^3$.
7. $b^3c^3 + r^3$.
8. $(2a)^3 - b^3$.
9. $8a^3 - c^3$.
10. $m^3 + 8n^3$.
11. $a^3b^3 + 8$.
12. $8y^3z^3 + 1$.
13. $(3x)^3 - y^3$.
14. $x^3 + (3)^3$.
15. $1 + 27d^3$.
16. $27a^3 - 1$.
17. $(a^2)^3 - b^3$.
18. $x^3 - y^6$.
19. $r^3 + 27$.
20. $a^3 - 64$.
21. $a^3 + 64$.
22. $x^3 - 64y^3$.
23. $a^3b^3 + c^3d^3$.
24. $64 - x^3$.
25. $a^3 - 125$.

596. The following supplementary exercises are applications of the preceding cases. Sometimes it is possible to apply more than one case to an exercise.

1. Factor $2a^4x - 2a^2x^3 - 24x^5$.

First divide by $2x$.

$$2a^4x - 2a^2x^3 - 24x^5 = 2x(a^4 - a^2x^2 - 12x^4),$$

but $$a^4 - a^2x^2 - 12x^4 = (a^2 + 3x^2)(a^2 - 4x^2);$$

hence $$2a^4x - 2a^2x^3 - 24x^5 = 2x(a^2 + 3x^2)(a^2 - 4x^2),$$

but $$a^2 - 4x^2 = (a + 2x)(a - 2x);$$

hence $$2a^4x - 2a^2x^3 - 24x^5 = 2x(a^2 + 3x^2)(a + 2x)(a - 2x).$$

2. Factor $x^6 - y^6$.

Since $x^6 = (x^3)^2$, and $y^6 = (y^3)^2$,

$$x^6 - y^6 = (x^3 + y^3)(x^3 - y^3).$$

Factoring the second member of the above equation,

$$x^6 - y^6 = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2).$$

When in doubt prove your work by either multiplication or division.

597. Factor:

1. $7x^3y + 21xyz^2$.
2. $ab^2 - 2abc + ac^2$.
3. $a^2b + 3abc + 2bc^2$.
4. $a^3 + 2a^2 + a$.
5. $x^3 - 216$.
6. $a^2b^2c^2 + 6abc + 9$.
7. $3n^4 + 81n$.
8. $n^4 - 81n$.
9. $a^4 - a$.
10. $2a^2 - 2ab - 24b^2$.
11. $x^4 + 2x^2y^2 + y^4$.
12. $x^5 + 5x$.
13. $y^5 - 25y^3$.
14. $m^2 + 25mn + 100n^2$.
15. $a^2 - 20abc + 100b^2c^2$.
16. $x^2 - 16y^4$.
17. $a^4 - 16a^2b + 55b^2$.
18. $x^3 + x^3y^3$.
19. $x^4 + x^2y^2$.
20. $x^4 - x^2y^2$.
21. $xy^2 - xy - 6x$.
22. $x^2y - 5xy^2 + 6y^3$.
23. $a^3b^2 + 5a^2b^3 + 10ab^5$.
24. $(3m)^2 + 2(3m) + 1$.
25. $(x+y)^2 + 2(x+y) + 1$.
26. $(x+y)^2 - 2(x+y)z + z^2$.
27. $3b^5 - 5b^3$.
28. $(x+y)^2 - 1$.
29. $a^2 - 2ab + b^2 - 1$.
30. $z^2 - z^5$.
31. $125x + x^4$.
32. $x^4 - 5x^2 + 4$.
33. $a^4 - 2a^2 - 8$.
34. $100 + 25y^2$.
35. $100 - 25y^2$.
36. $ab^2 - 144ac^2$.
37. $12xy^2 - 144xz^2$.
38. $3x^2 - 9x - 84$.
39. $ax^2 - a + 2abx + ab^2$.
40. $x^4 + 2x^2 + 1 - a^2$.
41. $x^4 + 2x^2 + 1 - x^2$.
42. $a^4 + a^2 + 1$.

    (Change 42 to same form as 41.)

43. $y^2 - 13y + 36$.
44. $b^4 - 13b^2 + 36$.
45. $z^5 + z^3 + z$.
46. $m^4 - 16$.
47. $16 - m^4n^6$.
48. $16m + 2m^4$.
49. $16a - a^4$.
50. $a^6 - 64$.

FRACTIONS.

598. Preliminary Exercises.

1. Reduce $\frac{xy}{xz}$ to its lowest terms. *Ans.* $\frac{y}{z}$.

Divide the numerator and the denominator by x.

2. Change $\frac{a}{b}$ to an equivalent fraction whose denominator is bc. *Ans.* $\frac{ac}{bc}$.

Multiply the numerator and the denominator by c.

The value of a fraction is not changed when both numerator and denominator are either multiplied or divided by the same quantity.

599. Oral Exercises.

Reduce:

1. $\frac{2\,ax}{3\,x}$.
2. $\frac{2\,ab}{3\,a^2}$.
3. $\frac{7\,b^3x^2}{21\,b^2x^3}$.
4. $\frac{7\,a^3}{a^3}$.
5. $\frac{ax+ay}{3\,ab}$.
7. $\frac{2\,ab^2}{a+ab}$.
9. $\frac{nxy+x}{ax+x^2}$.
6. $\frac{12\,a^2b^3}{6\,ab}$.
8. $\frac{a^2+2\,ab}{3\,ab}$.
10. $\frac{2\,px+3\,xy}{px+xy}$.

600. Sight Exercises.

Give answers at sight:

1. $\frac{b}{3}=\frac{?}{3\,x}$.
2. $\frac{2\,y}{3\,x}=\frac{?}{3\,x^2}$.
3. $\frac{2\,a}{3}=\frac{2\,ac}{?}$.
4. $\frac{5\,xy}{7\,m}=\frac{?}{14\,mn}$.
5. $4\,a=\frac{?}{a}$.
6. $4\,a=\frac{16\,a^3}{?}$.
7. $\frac{y+z}{2\,a}=\frac{?}{2\,ab}$.
8. $2\,ab^2=\frac{2\,ab^3}{?}$.
9. $3\,mt=\frac{?}{3\,mt}$.
10. $a+b=\frac{2\,a^2b+2\,ab^2}{?}$

601. Written Exercises.

1. Reduce $\frac{x^2+11x+30}{x+5}$.

Divide the numerator by the denominator. *Ans.* $x+6$.

2. Reduce $\frac{x^2+12x+35}{x+5}$.

3. Reduce $\frac{x^2+10x+20}{x+4}$. *Ans.* $x+6-\frac{4}{x+4}$.

4. $b+c=\frac{?}{b+c}$.

5. $\frac{1}{c^2+c+5}=\frac{2c+3}{?}$.

6. Change $x+2$ to a fraction whose denominator is $x+3$.

$x+2=\frac{x+2}{1}$. Multiply the numerator and the denominator by $x+3$.

7. $x-3=\frac{?}{x+5}$.

8. $x^2-x+1=\frac{?}{x+1}$.

9. Change $x+\frac{x-3}{x+2}$ to a fraction.

$$x=\frac{x^2+2x}{x+2}.$$

$$\frac{x^2+2x}{x+2}+\frac{x-3}{x+2}=\frac{?}{x+2}.$$

What may be done with the numerators when the denominator is common?

10. Reduce $\frac{8x^3-1}{2x-1}$.

11. $x-1-\frac{x-3}{x+2}=\frac{?}{x+2}$.

12. $\frac{3}{2x}+\frac{5}{x+2}=\frac{?}{2x^2+4x}$.

13. $\frac{x}{x+2}+\frac{3x}{x+2}=?$

14. $\frac{1}{x}=\frac{?}{2x^2+x}$.

15. $\frac{2}{y}=\frac{?}{y^2-y}$.

16. $\frac{3}{x}+\frac{x-3}{2x+1}+\frac{x^2-5}{2x^2+x}=\frac{?}{2x^2+x}$.

17. $\left(\frac{x+1}{x-1}\right)\left(\frac{x+2}{x-2}\right) = ?$; *i.e.*, $\frac{(x+1)(x+2)}{(x-1)(x-2)} = ?$

$\frac{2}{3}$ multiplied by $\frac{5}{7} = ?$

18. $\frac{x+1}{x-1} \div \frac{x+2}{x-2} = ?$; *i.e.*, $\frac{(x+1)(x-2)}{(x-1)(x+2)} = ?$

19. By what quantity must $x-5$ be multiplied to give a product of x^2+x-30?

By what number must 7 be multiplied to give a product of 63?

20. $\frac{1}{x-5} = \frac{?}{x^2+x-30}$.

21. $\frac{a}{x-6} = \frac{?}{x^2-x-30}$.

22. $\frac{a}{x-1} = \frac{ax+a}{?}$.

23. $\left(\frac{x-2}{x-5}\right)\left(\frac{x+5}{x+2}\right) = ?$

24. $\frac{x+1}{x-1} = \frac{x^2+2x+1}{?}$.

25. $\frac{x-5}{x+1} = \frac{?}{x^2-1}$.

26. $\frac{x-5}{x+3} \div \frac{x-1}{x+4} = ?$

27. Add $\frac{x-5}{x+1}$ and $\frac{x-2}{x-1}$. *Ans.* $\frac{2x^2-7x+3}{x^2-1}$.

28. $\frac{x-2}{x-1} - \frac{x-5}{x+1} = ?$

29. $\frac{x+5}{x-3} = \frac{?}{x^2-x-6}$.

30. $\frac{27a^3x^3-2}{3ax-1} = ?$

PURE QUADRATICS.

602. Given $\frac{x^2+6}{5}=\frac{3x^2-66}{9}$, to find the value of x.

Clearing of fractions,	$9x^2+54=15x^2-330.$
Transposing and combining,	$-6x^2=-384.$
Dividing by -6,	$x^2=64.$
Extracting square root,	$x=\pm 8.$

Since $(-8)\times(-8)=64$, the square root of 64 may be either $+8$ or -8. It is written ± 8, and is read "*positive or negative* 8." (It is sometimes less correctly called *plus or minus* 8.)

603. Written Exercises.

Find value of x, y, z, etc.:

1. $x^2-13=36.$
2. $3y^2+25=100.$
3. $5z^2-13=3z^2+37.$
4. $5(x^2+17)-3x^2+63=198.$
5. $5(x^2+17)-3(x^2-21)=198.$
6. $y^2+2y+1-y^2=49.$
7. $(x+1)^2-x^2=49.$
8. $\frac{y^2+5}{3}-\frac{2y^2-18}{4}=2.$
9. $\frac{z+7}{z-3}=\frac{z-5}{z-9}.$
10. $\frac{20x}{x-1}=\frac{30x}{x+1}.$

11. $(x-3)(x+3)=40.$

12. $(x+5)(x+5)=10x+26.$

13. $(x+4)^2=8x+80.$

14. $z^2+64=5z^2.$

15. $3x^2+18=2\frac{1}{2}x^2+36.$

16. $(x-3)^2-(x-5)^2=12.$

17. $(x+7)(x-9)=(x-3)(x-5).$

18. $\frac{x}{4}+\frac{4}{x}=\frac{x}{9}+\frac{9}{x}.$

19. $\frac{x+7}{x-5}=\frac{x-3}{x-9}.$

20. $\frac{y-9}{y-5}=\frac{y-3}{y+7}.$

604. Written Problems.

1. Find the dimensions of a field, the length of which is twice its breadth, its area being 1800 square rods.

2. The surface of the six equal faces of a cube contains 96 square inches. Find the length of one edge.

3. One number is four-fifths of another, and their product is 80. What are the numbers?

4. One-third of a number multiplied by two-fifths of the same number gives a product of 270. Find the number.

5. Thirty per cent of a number multiplied by forty per cent of the same number gives a product of 300. What is the number?

6. Thirty per cent of twenty per cent of a number is 300. What is the number?

7. The base of a right-angled triangle is $\frac{3}{4}$ as long as the perpendicular, and the area of the triangle is 96 square rods. Find the length of the base. What is the length of the hypotenuse?

8. The base of a right-angled triangle measures x yards, the perpendicular measures $\frac{3x}{4}$ yards. What is the length of the hypotenuse? If the hypotenuse measures 15 yards, find the length of the base.

9. The base of a right-angled triangle measures x feet, the hypotenuse measures $(x+9)$ feet, the perpendicular measures 15 feet. What is the length of the base?

10. The difference between the squares of two consecutive numbers is 49. What are the numbers?

11. The difference between two numbers is 6. The sum of their squares is 146. What are the numbers?

Let $x-3$ = smaller number,
and $x+3$ = greater number.

AFFECTED QUADRATICS.

605. Preliminary Exercises.

$$(x+1)(x+1) = x^2+2x+1.$$

Compare with Article 582.

$$(x-1)(x-1) = x^2-2x+1.$$

Compare with Article 584.

$$(a+b)^2 = a^2+2ab+b^2.$$

$$(m-n)^2 = m^2-2mn+n^2.$$

$$(10+5)^2 = 10^2+2\times10\times5+5^2.$$

$$(10-3)^2 = 10^2-2\times10\times3+3^2.$$

606. Oral Exercises.

Square:

1. $x+3$.	4. $x+10$.	7. $30-1$.	10. $x-y$.
2. $x-7$.	5. $a-b$.	8. $40-1$.	11. $80+5$.
3. $x-9$.	6. $x+y$.	9. $m+n$.	12. $60-5$.

607. Oral Exercises.

Extract the square root of

1. x^2+6x+9.	6. $x^2+2xy+y^2$.
2. $x^2-14x+49$.	7. $x^2-2xy+y^2$.
3. $x^2-18x+81$.	8. $a^2-2ab+b^2$.
4. $x^2+20x+100$.	9. $x^2-24x+144$.
5. $a^2+2ab+b^2$.	10. $x^2+22x+121$.

The square of $(x+3)$ consists of how many terms? Of how many terms does $(x+4)^2$ consist? $(x+5)^2$?

608. Supply term necessary to make a complete square:

1. $x^2+6x+?$	6. $x^2+2x+?$
2. $x^2-12x+?$	7. $x^2-4x+?$
3. $x^2-8x+?$	8. $x^2-10x+?$
4. $x^2-16x+?$	9. $x^2+14x+?$
5. $x^2+18x+?$	10. $x^2-22x+?$

609. Written Exercises.

Given $x^2+6x=27$.

What number must be added to the first member of the equation to make it a "complete" square?

If a number is added to one member of an equation, what must be done to the other member to preserve the equality?

610. Extract the square root of both members of the following equations, adding to both, where necessary, such a number as will make the first member a complete square.

1. $x^2 + 6x + 9 = 40 + 9$. 2. $x^2 - 12x + 36 = 28 + 36$.

Remember that $(+7) \times (+7) = 49$, and that $(-7) \times (-7) = 49$.
$\therefore \sqrt{49} = +7$ or -7, written ± 7.

3. $x^2 - 8x + 16 = 20 + 16$.
4. $x^2 - 16x + 64 = -39 + 64$.
5. $x^2 + 18x + ? = 19 + ?$
6. $x^2 + 2x + ? = 24 + ?$
7. $x^2 - 14x = 15$.
8. $x^2 - 22x = 23$.
9. $x^2 + 14x = 51$.
10. $x^2 - 22x = 48$.

611. Given $x^2 - 10x = 24$.

Completing the square, we have $x^2 - 10x + 25 = 24 + 25 = 49$. Extracting the square root of both sides, we have

$$x - 5 = \pm 7,$$

$$x = 7 + 5 = 12, \text{ or } x = -7 + 5 = -2.$$

Ans. 12 or -2.

612. Written Exercises.

Find values of x:

1. $x^2 - 6x = 7$.
2. $x^2 - 12x = 108$.
3. $x^2 + 2x = 48$.
4. $x^2 + 18x = 115$.
5. $x^2 - 14x = -13$.
6. $x^2 - 10x = 0$.
7. $x^2 + 20x = 125$.
8. $x^2 + 26x = 56$.
9. $x^2 - 24x = 0$.
10. $x^2 - 8x = 384$.
11. $x^2 - 4x = -3$.
12. $x^2 + 30x = 175$.
13. $x^2 + 28x = 29$.
14. $x^2 + 22x = 104$.
15. $x^2 - 16x = -64$.
16. $x^2 + 36x = 76$.

To make the first member a complete square, you added the square of what part of the coefficient of x?

613. Written Exercises.

Find values of x:

1. $x^2+x=12$,
   $x^2+x+(\frac{1}{2})^2=12+(\frac{1}{2})^2$.
2. $x^2-3x=10$,
   $x^2-3x+(\frac{3}{2})^2=10+(\frac{3}{2})^2$.
3. $x^2+5x=-4$.
4. $x^2-7x=8$.
5. $x^2+9x=-20$.
6. $x^2-11x=-28$.
7. $x^2+13x=-42$.
8. $x^2-15x=76$.
9. $x^2-17x=18$.
10. $x^2+19x=-18$.

614. When x^2 has a coefficient, divide both members by the coefficient.

$$3x^2+9x=84.$$

Dividing by 3, $x^2+3x=28$.

Completing the square,

$$x^2+3x+(\tfrac{3}{2})^2=28+\tfrac{9}{4}=\frac{112+9}{4}=\frac{121}{4}.$$

Extracting square root, $x+\frac{3}{2}=\pm\frac{11}{2}$.

$\therefore x=\frac{11}{2}-\frac{3}{2}=\frac{8}{2}=4$; or $-\frac{11}{2}-\frac{3}{2}=-\frac{14}{2}=-7$.

Ans. 4 or -7.

615. Written Exercises.

1. $6x^2-6x=36$.
2. $9x^2+9x=180$.
3. $7x^2+28x=147$.
4. $4x^2-40x=-64$.
5. $8x^2-16x=504$.
6. $3x^2+9x=54$.
7. $8x^2-72x=-160$.
8. $7x^2+49x=56$.
9. $3x^2+21x=54$.
10. $5x^2-25x=-20$.

616. Five times nothing = ?

Zero multiplied by one million = ?

If $x=5$,

$x-5=?$

$10(x-5)=?$

$(x+5)(x-5)=?$

If one of two factors is zero, the product is zero.

The converse is also true.

If the product of two factors is zero, one of the factors is zero.

Given $(x-2)(x-3)=0.$

One of the factors in the above equation is equal to zero.

If $x-2=0,$

by transposing we get $x=2.$

If $x-3=0,$

$x=3.$

617. A quadratic equation may sometimes be readily solved by factoring.

1. $x^2-5x=-6.$

$x^2-5x+6=0.$

$(x-3)(x-2)=0.$

$x=3$ or $2.$

2. $x^2-5x=14.$

$x^2-5x-14=0.$

$(x-7)(x+2)=0.$

$x=7$ or $-2.$

Solve by factoring:

3. $x^2+x-6=0.$

4. $x^2+2x-3=0.$

5. $x^2-3x+2=12.$

6. $y^2+7y+15=3.$

7. $x^2-7x+20=8.$

8. $z^2-4z+7=19.$

9. $y^2+10=28+3y.$

10. $x^2-2x-24=0.$

11. $x^2-15x=16.$

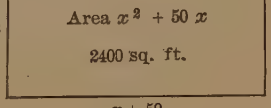
12. $y^2+19y=20.$

618. Written Problems.

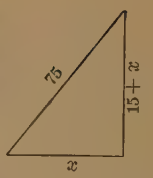
1. The sum of two numbers is 12; their product is 32. What are the numbers?

x and $12-x=$ numbers. $(12-x)x=$ product.

2. The base of a rectangle is 50 feet longer than its altitude. Its area is 2400 square feet. How long is the base?

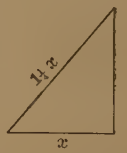


3. The perpendicular of a right-angled triangle measures 15 yards more than the base. The hypotenuse is 75 yards. Find the length of the perpendicular.



$$x^2 + (15 + x)^2 = 75^2.$$

4. The hypotenuse of a right-angled triangle is $1\frac{1}{4}$ times as long as the base. The area of the triangle is 150 square yards. How long is the hypotenuse?

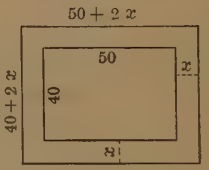


Perpendicular $= \sqrt{(\frac{5}{4}x)^2 - x^2}$; area $= \frac{1}{2}$ base $\times$ perpendicular.

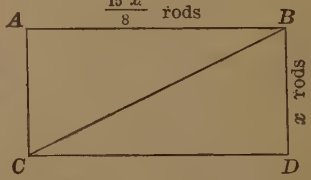
5. The entire surface of a square prism is 170 square feet. Its altitude is 6 feet, and one side of its base is x feet. Find the value of x.

6. A garden 50 feet long, 40 feet wide, has a walk just outside it x feet wide. Find the area of the walk.

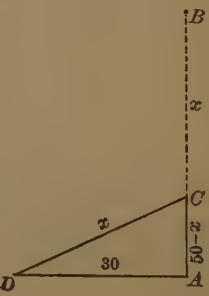
If the area of the walk is 784 square feet, what is its width?



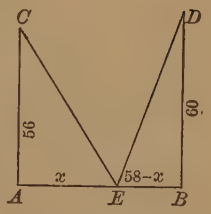
7. A field, *ABCD*, contains 12 acres. Its length is $1\frac{7}{8}$ times its breadth. How many rods long is the diagonal *BC*?



8. A flag-staff, *AB*, 50 feet high, was broken off at the point *C*. The broken part, resting on *C*, reached the ground *D*, 30 feet from the base of the staff. Find the length of the part broken off.

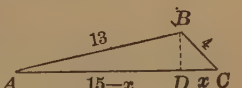


9. A ladder, CE or DE, placed at a point E, in a street 58 feet wide between the opposite houses, just touches the top of a house, DB, 60 feet high on one side of the street, or the top of a house, CA, 56 feet high on the other side. Find the length of the ladder.



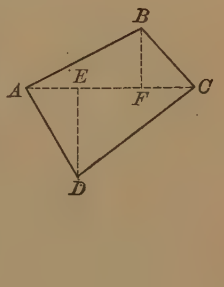
$$\overline{DE}^2 = 60^2 + (58 - x)^2 = \overline{CE}^2 = 56^2 + x^2.$$

10. ABC is a triangle. The side AB measures 13 feet; the side BC, 4 feet; AC, 15 feet. Find the altitude BD.



$$\overline{BD}^2 = \overline{AB}^2 - \overline{AD}^2 = \overline{BC}^2 - \overline{CD}^2.$$

11. $ABCD$ is a trapezium. $AB = 34$ feet; $BC = 20$ feet; $CD = 40$ feet; $DA = 26$ feet. The perpendicular BF measures 16 feet. Find the length of the diagonal AC and of the perpendicular ED.



CHAPTER VIII.

GEOMETRY.

619. Vertical Lines.

Hang a weight from a fixed point by a string. When the weight stops swinging the string is in a *vertical* line. What way does the lower end of the string point? the higher end? Hold a sheet of ruled paper so that the lines are vertical.

620. Oblique and Horizontal Lines.

Hold a pointer so that it points upward but not straight up. It is in an *oblique* line.

Hold a pointer so that it does not point or slant either up or down. It is in a *horizontal* line.

NOTE. — In representing vertical, horizontal, or oblique lines on the page of a book or a sheet of paper it is assumed that the book or paper is held in an upright position.

621. Oral Exercises.

1. What kind of line is represented by the course of a drop of water running down a roof?

2. By the course of a falling raindrop when there is no wind?

3. By the course of a falling raindrop when there is a wind?

4. By straws floating on the surface of still water?

Use the object for the four following exercises.

5. How many lines are there in the edges of a cube or a rectangular box?

6. When the cube is placed on a level table, how many edges are vertical? How many are horizontal? How many are oblique?

7. Hold the cube so that four edges are horizontal. How many are vertical? How many are oblique?

8. Hold the cube so that no edges are horizontal. How many are oblique? How many are vertical?

9. A straight line is 3 feet long. What kind of line is it if one end is 4 feet from the floor and the other end is 1 foot from the floor?

10. If one end of a 3-foot straight line is 4 feet from the floor and the other end is 2 feet from the floor, what kind of line is it?

11. If each end of a straight line is 5 feet from the floor, what kind of line is it?

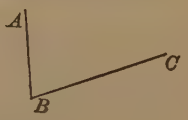
12. If one end of a straight line is 4 feet from the floor and the middle is 4 feet from the floor, what kind of line is it?

13. A vertical straight line is 5 feet long. The middle is 5 feet from the floor. How far is each end from the floor?

14. A vertical line is 4 feet long. One end is 5 feet from the floor. How far from the floor is the other end? Why are there two answers?

622. Angles.

When the ends of two straight lines meet they form an angle.



What two lines form the angle ABC in the above figure?

At what point do they meet?

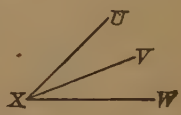
The point B is the vertex of the angle ABC.

What is the vertex of an angle?

623. Designation of Angles.

The angle formed by the lines ST and TU may be called the angle T. It is frequently better to call it the angle STU or UTS, the letter at the vertex being placed between the two others.

The use of the three letters is necessary where two or more angles have vertices at the same point, as in the accompanying figure, where UX, VX, and WX meet at the point X.



624. Exercises.

Draw a horizontal line 3 inches long. Mark a point in this line one inch from the left end. From this point draw a line upward slanting towards the right. Mark each end of each line by a letter. How many angles have you formed? Designate each of these angles by three letters.

NOTE. — The above exercise may be varied for blackboard drill — draw a vertical line 11 inches long; mark a point 4 inches from the top; draw a line to the left slanting downward, etc.

How many angles are formed when two lines meet at their ends? When two lines pass *through* the same point? When from a point in one line another line is drawn?

625. Circular Measure.

60 seconds ($''$)	1 minute.
60 minutes ($'$)	1 degree.
360 degrees ($^\circ$)	1 circle, or circumference.

626. Exercises.

1. What part of a circumference is 180°? 90°? 60°? 30°? 45°? 36°? 72°?

2. 1° on the circumference of a circle is 5 inches. What is the length of the circumference?

3. The circumference of a circle is 9000 feet. $1° = ?$ $1' = ?$

4. How many degrees are there between the XII and the I on the face of a clock? between the XII and VI? between the XII and III? between the III and VII?

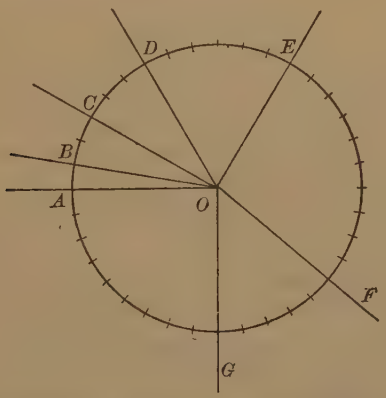
5. If one degree of the earth's circumference is $69\frac{1}{6}$ miles, find the circumference.

6. Through how many degrees does the minute hand of a clock pass in 1 hour? in $\frac{1}{2}$ hour? in 15 minutes? in 5 minutes? in 10 minutes? in 1 minute? in 3 minutes?

627. Angular Measure.

The angle at the centre of a circle has the same number of degrees as the arc between the sides of the angle.

Thus, in the following figure the angle AOC has the same number of degrees as the arc ABC.

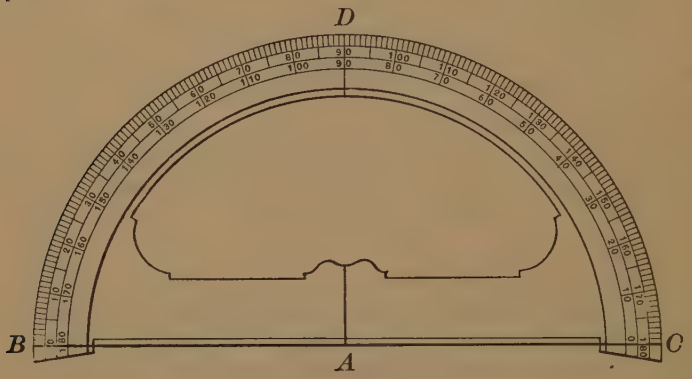


The circumference of this circle is divided into 36 equal parts. How many degrees are there in each part? How many degrees are there in each of the following angles?

AOB, BOC, COD, DOE, EOF, FOG, GOA, AOE, DOF.

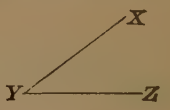
628. The Protractor.

The number of degrees in an angle may be measured by a *protractor.*



SEMICIRCULAR PROTRACTOR

To measure an angle, XYZ, for instance, produce the lines YX and YZ. Place the point A of the protractor on the vertex (Y) of the angle, and the edge AC on the line YZ produced. Using the lower line of figures, read off from the protractor the number of degrees at the point where the line YX produced cuts the semicircle.



In measuring the angle DEF, the line AB is placed on EF, the point A on the vertex E. The number of degrees in this case is read from the upper row of figures.

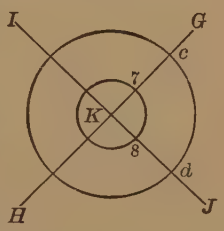
NOTE. — There is only one point on the protractor where the numbers of the upper and lower lines of figures are equal. What is the number of degrees at that point? What kind of angle is measured at that point? If an angle is acute, would you read its measure by the larger or by the smaller number?

EXERCISES IN CONSTRUCTION.

629. NOTE. — In the following exercises, the ruler, the compasses, and the protractor may be used.

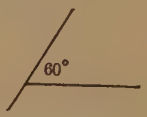
The drawing should be carefully done with a sharp, hard pencil.

1. Draw an obtuse angle formed by two lines, each one inch long. Draw an acute angle formed by two lines, each six inches long. Which is the larger?



2. The lines GH and IJ intersect at K, making four right angles. Which arc is longer, 7 8 or cd? Which contains the greater number of degrees?

3. Draw two lines meeting at an angle of 45°. Two lines meeting at an angle of 90°. Two meeting at an angle of 135°.



4. Draw two lines making two angles, one of which measures 60°. How many degrees does the other angle contain?

5. To a horizontal line draw a line making two equal adjacent angles. How many degrees does each angle contain?

Two angles are said to be adjacent when they have one side in common.

To a vertical line draw a line making two equal adjacent angles. How many degrees does each angle contain?

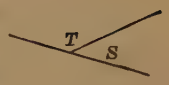
To an oblique line draw a line making two equal adjacent angles. How many degrees does each angle contain?

6. How many degrees are there in a right angle?

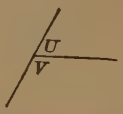
7. To an oblique line draw a line making two unequal adjacent angles. How many degrees are there in the sum of the two angles?

Two angles are said to be *supplementary* when they are together equal to two right angles.

8. How many degrees in the angle T, if S contains 75°?



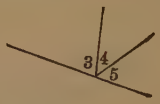
V measures 110°. How many degrees does U measure?



If one of two supplementary angles measures $63\frac{1}{2}°$, how many degrees are there in the other angle?

How many degrees are there in an angle supplementary to one of 47° 45′?

9. Construct angle 5, 60°; angle 4, 50°. Measure angle 3.



How many degrees and minutes will there be in angle 5 when 3 contains $49\frac{1}{2}°$ and 4 contains $83\frac{3}{4}°$?

When angle 3 contains 36° 30′ and angle 5 contains 79° 45′, how many degrees and minutes will angle 4 contain?

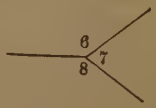
10. Erect a perpendicular at each extremity of a horizontal line. At each extremity of a vertical line. At each extremity of an oblique line.

Note. — A line making a right angle with another line is said to be *perpendicular* to it.

11. Construct a square upon a horizontal line. Upon an oblique line.

12. Draw two lines intersecting at an angle of 100°. Mark in each of the other three angles the number of degrees it contains.

13. Draw two lines making an angle (6) of 150°. Construct an adjacent angle (7) containing 80°. How many degrees will angle 8 contain?



14. How many degrees will there be in the sum of five angles having the same vertex?

15. Draw five equal angles having a common vertex.

16. Draw six equal angles having a common vertex. Is any angle supplementary to the angle next it? Why?

Are any of the angles vertical? Why?

17. Draw two angles, one of 65° and the other of 25°. Draw a third angle equal to the sum of both.

Draw an angle equal to their difference.

18. Draw an angle equal to the sum of three angles measuring, respectively, 40°, 50°, and 60°.

630. Parallels.

Lines which lie in the same plane and which cannot meet, no matter how far produced, are said to be *parallel.*

19. Using the protractor, draw two or more lines that shall be perpendicular to a horizontal line. Where will they meet?

Draw two or more that shall be perpendicular to a vertical line. Where will they meet?

Draw two or more that shall be perpendicular to an oblique line. Where will they meet?

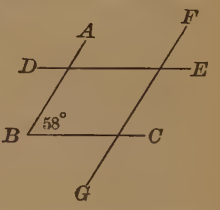
20. To a horizontal line draw two or more lines running in the same direction, and each making an angle of 35° with the first line. Will the oblique lines meet?

Draw two or more lines running in the same direction, and each making an angle of 125° with a vertical line. Will the oblique lines meet if produced very far?

Draw two or more lines running in the same direction, and each making an angle of 74° with an oblique line. Will the former lines meet?

21. Draw two lines making angles of 30° and 60°, respectively, with a third line. Will the two former lines meet if produced in either direction?

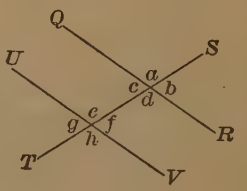
22. Draw a line, AB, meeting a horizontal line, BC, at an angle of 58°. Draw a third line, DE, parallel to the horizontal line, and cutting the oblique line. What angles does it make with the oblique line?



Draw a fourth line, FG, parallel to the oblique line, and cutting both horizontal lines.

Mark in each of the twelve angles the number of degrees it contains.

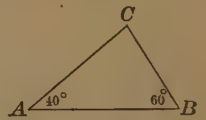
23. QR and UV are parallel lines, cut by a line ST. If the angle b measures 50°, how many degrees does a measure?



Find the number of degrees in each of the other six angles.

631. Triangles.

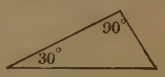
24. From the extremities of the line AB, draw lines that shall make angles of 60° and 40°, respectively, with AB. Prolong the lines until they meet at C, forming a triangle.



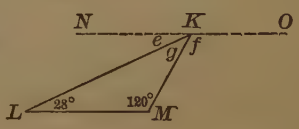
Measure the angle at C. How many degrees does it contain? How many degrees are there in the sum of the three angles of the triangle?

25. Construct a triangle having one angle of 90° and one of 30°. Measure the third angle.

How many degrees are there in the sum of the three angles?



26. Construct a triangle, KLM, making the angles at the base 28° and 120°, respectively. Draw NO, parallel to LM.



Is the angle e equal to any angle of the triangle? How many degrees does it contain? Is the angle f equal to any angle of the triangle? How many degrees does it contain?

How many degrees are there in the sum of the angles e, g, and f? How many degrees are there in the angle g?

27. How many degrees are there in the three angles of any triangle?

28. Two angles of a triangle measure 36° and 65°, respectively. How many degrees does the third angle contain?

29. Draw a triangle containing two angles of 50° and 70°, respectively. How many degrees are there in the third angle?

Measure each side, and mark on the side its length.

Opposite which angle is found the longest side? Opposite which, the shortest side?

30. Draw a triangle having two angles of 75° each. Are any two of its sides equal?

Draw a triangle having two angles of 50° each. Are any of its sides equal?

31. Draw a triangle having two angles of 60° each. How many degrees does the third angle contain?

Are any of its sides equal?

32. If a triangle has two of its sides equal, what is true of its angles?

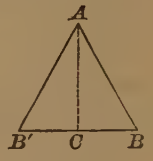
33. If a triangle has three of its sides equal, what is true of its angles?

632. A triangle having all its sides equal, is called an *equilateral* triangle.

A triangle having two equal sides, is called an *isosceles* triangle.

A triangle having all its sides unequal, is called a *scalene* triangle.

34. How does a perpendicular let fall upon the base of an isosceles triangle from the opposite angle divide the angle? How does it divide the base? How do the angles at the base of an isosceles triangle compare with each other as to size?



The *unequal* side of an isosceles triangle is called the base.

35. Draw an isosceles triangle having the base a vertical line.

An isosceles triangle having the vertex below the base.

One having an oblique line for the base.

36. Draw a right-angled isosceles triangle. How many degrees will there be in each of the other angles?

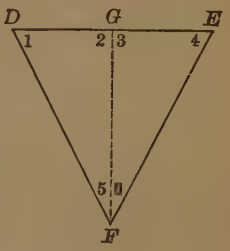
Draw an obtuse-angled isosceles triangle.

37. How many degrees will there be in each angle of an equilateral triangle?

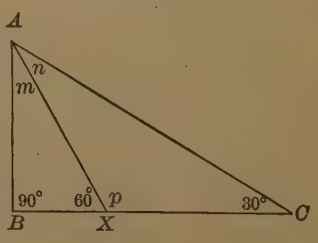
Draw an equilateral triangle having one side vertical.

Draw an equilateral triangle having its vertex below the base.

38. *DEF* is an isosceles triangle, *DF* and *EF* being the equal sides. If the angle 1 measures 50°, how many degrees are there in each of the other five angles, when the line *FG* bisects the base?



39. *ABC* is a right-angled triangle, the angle at *B* measuring 90°, and the angle at *C* measuring 30°. If the line *AX* is so drawn as to make the angle *AXB* equal to 60°, find the number of degrees in the angles *m*, *n*, and *p*, respectively.



633. Quadrilaterals.

A plane figure of four sides is called a *quadrilateral.*

When the opposite sides are parallel, the quadrilateral is called a *parallelogram.* (Figs. 1 to 8.)

A *rectangle* is a parallelogram all of whose angles are right angles. (Figs. 1 to 4.)

When the four sides of a rectangle are equal to each other, it is called a *square.* (Figs. 1 and 2.)

The term *oblong* is frequently applied to rectangles whose adjacent sides are unequal. (Figs. 3 and 4.)

FIG. 1. FIG. 2. FIG. 3. FIG. 4.

A *rhombus* is a parallelogram all of whose sides are equal, but whose angles are oblique. (Figs. 5 and 6.)

When the adjacent sides of a parallelogram are unequal and the angles are oblique, it is called a *rhomboid.* (Figs. 7 and 8.)

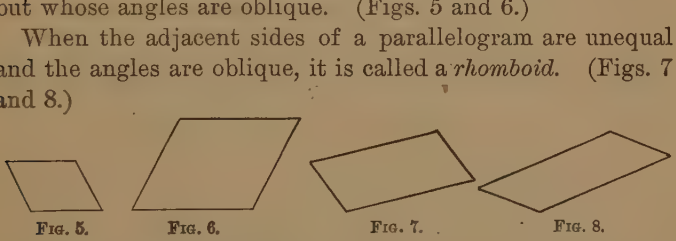


FIG. 5. FIG. 6. FIG. 7. FIG. 8.

A *trapezoid* is a quadrilateral having only two of its sides parallel. (Figs. 9 and 10.)

A *trapezium* is a quadrilateral having no two sides parallel. (Figs. 11 and 12.)

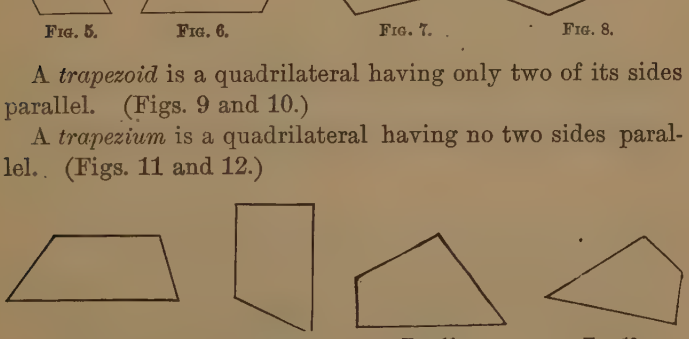
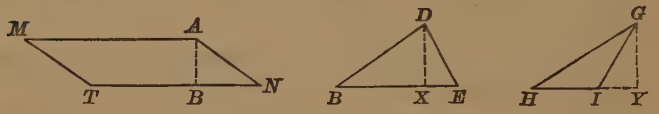


FIG. 9. FIG. 10. FIG. 11. FIG. 12.

634. The *altitude* of a *parallelogram* is the perpendicular distance between its base and the side opposite.



The *altitude* of a *triangle* is the perpendicular distance between the vertex and the base, or between the vertex and base produced.

AB is the altitude of *MANT*; *DX* is the altitude of *DBE*; *GY* of *GHI*.

40. Draw a parallelogram. How many angles does it contain? Into how few triangles can you divide a parallelogram? How many degrees are there in the sum of the angles of each triangle? How many degrees are there in the sum of the angles of a parallelogram?

41. Construct a parallelogram, the adjacent sides of which shall measure 2 inches and 3 inches, respectively, and the angle between them 60°. How long will each of the other two sides be? Measure each of the other angles. How many degrees are there in the sum of the four angles?

42. Construct a trapezoid having a base of 5 inches, altitude 3 inches, the angles at the base measuring 90° and 60°, respectively. Measure the remaining angles, and find the sum of the four angles. How long is each of the remaining sides?

43. Fold a piece of paper twice at right angles, and cut off the folded corner, making a rhombus when the part cut off is opened out.

Can you cut out a rhombus having two angles of 60° each? A rhombus having two angles of 80° each?

44. Can you so cut a piece of paper, folded twice at right angles, that the part cut off will be a square?

45. Draw a rectangle, base $2\frac{1}{2}$ inches, altitude 2 inches.
A rhomboid, base $2\frac{1}{2}$ inches, altitude 2 inches.

46. Make, out of paper, a rectangle and a rhomboid, each having the above dimensions, and endeavor to ascertain, by cutting, whether or not they are equal to each other in area.

635. The Circle.

47. Draw a circle. Between two points on the circumference draw a line that does not pass through the centre.

This line is called a *chord*.

48. Draw a circle. In it draw two diameters, a radius, and three chords. Write on each line its name.

49. Draw a part of the circumference of a circle greater than one-half of it. Draw the chord.

A part of the circumference is called an *arc*.

50. Draw an arc less than a semi-circumference. Draw a chord. Write the name on each.
Can you make a chord that will be longer than the diameter?

51. Draw two equal circles. In the first draw the chord of an arc of 120°. In the second, the chord of an arc of 240°. What is the ratio between the two chords you have drawn?

52. In a circle draw a chord equal in length to the radius. How many degrees are there in the arc whose chord has been drawn?

53. Draw an arc of 72°. To its extremities draw two radii.

The part of the surface of a circle enclosed by two radii and the intercepted arc is called a *sector*.

54. Draw a sector of 60° (a *sextant*). A sector of 90° (a *quadrant*).

55. Draw an arc of 120°. Draw the chord.

The part of the surface of a circle bounded by an arc and its chord is called a *segment*.

56. Draw several circles having the same centre, but of unequal radii (*concentric* circles).

57. Draw two equal circles just touching each other (*tangent*). Draw two unequal circles tangent to each other.

Within a large circle draw a smaller one tangent to it.

58. Draw circles of equal radii cutting each other. Draw intersecting circles of unequal radii.

636. Pentagons, Hexagons, Octagons.

59. Divide the circumference of a circle into four equal arcs. Draw the chords, forming an inscribed square.

60. If you wish to inscribe in a circle a figure of five equal sides, into how many equal arcs must the circumference be divided? How many degrees will each arc contain?

637. A plane figure bounded by straight lines is called a *polygon*.

A five-sided polygon is called a *pentagon;* one of six sides, a *hexagon;* of seven, a *heptagon;* of eight, an *octagon;* of nine, a *nonagon;* of ten, a *decagon;* etc.

A *regular* polygon is one that is both equilateral and equiangular.

61. Inscribe a regular pentagon in a circle. Use the protractor.

62. Inscribe in a circle a regular hexagon. A regular octagon. An equilateral triangle.

63. Inscribe in a circle a regular hexagon. Connect the opposite corners by lines passing through the centre of the circle, forming six triangles.

How many degrees are there in each of the six angles about the centre of the circle? In each of the twelve angles at the circumference? How many degrees are there in the sum of angles 1 and 2?

Is each of the six triangles scalene, equilateral, or isosceles?

64. Divide a regular inscribed pentagon into five equal triangles by lines drawn from the centre of the circle.

What kind of triangles are formed; isosceles, scalene, or equilateral?

How many degrees are there in each angle at the centre? In each angle at the circumference? How many degrees are there in the sum of two adjoining angles at the circumference? In each angle of the pentagon?

65. About a circle circumscribe a square. An equilateral triangle. A regular pentagon. A regular hexagon. A regular octagon.

PROBLEMS IN CONSTRUCTION.

638. In drawing the following exercises, only the ruler and the compasses are to be used. Use neither the protractor nor the triangle.

66. Draw a circle, radius an inch and a half. Outside of it, and tangent to it, draw a second circle of an inch radius.

How far apart are the centres?

67. Draw two tangent circles having radii of an inch and a half and an inch, respectively, one within the other.

How long is the line joining the centres?

68. With centres 3 inches apart draw two equal circles tangent to each other. How long is the radius of each?

69. With centres three inches apart draw two equal circles of 2 inches' radius. Connect the centres.

Draw a line joining the two points in which the circles intersect. How does this line divide the line connecting the centres?

Draw radii from each centre to each point of intersection.

70. Construct an isosceles triangle, base 3 inches, equal sides 2 inches.

NOTE. — Use circles or arcs where necessary.

71. Construct an isosceles triangle, base $3\frac{1}{2}$ inches, equal sides 4 inches.

Divide it into two equal parts. Do not locate the centre of the base by measurements.

72. On a vertical line construct an isosceles triangle. Without measuring the length of the base draw a perpendicular to the centre of the base.

73. Bisect a vertical line. An oblique line.

Do not measure the length of the line.

74. Construct an equilateral triangle on a two-inch line.

75. Construct an equilateral triangle on a vertical line. On an oblique line.

76. Cut out two equal right-angled triangles. Put them together in different ways so as to form two different isosceles triangles.

77. Construct a scalene triangle.

A triangle having sides measuring 1, $1\frac{1}{2}$, 2 inches, respectively.

One whose sides measure 2, $2\frac{1}{2}$, and 3 inches, respectively.

78. Can you construct an isosceles triangle whose base measures 4 inches, equal sides 2 inches?

Try to construct a scalene triangle with sides measuring 1, 2, and 3 inches, respectively.

79. Draw a circle. In it draw a chord.

Bisect the chord, using as few lines and as short ones as you can.

NOTE. — Do not use the ruler to ascertain the length of the chord before bisecting it.

80. Divide a sector into two equal parts.

81. Draw a circle. Draw a chord. Draw a radius through the centre of the chord.

Is the radius perpendicular to the chord? Why?

82. Bisect the arc of a circle and its chord.

Bisect the arc of a circle without drawing the chord.

83. Draw a perpendicular to the middle point of a horizontal line. To the middle point of a vertical line. To the middle point of an oblique line.

84. Draw in a circle two diameters perpendicular to each other.

85. Divide the circumference of a circle into four equal parts. Into eight equal parts.

Inscribe a square in a circle.

86. Inscribe a regular octagon in a circle.

87. Connect the opposite vertices of a regular octagon inscribed in a circle by lines passing through the centre of the circle.

Lines connecting the opposite vertices of a polygon are called *diagonals*.

88. Inscribe a square in a circle. Circumscribe a square whose sides shall be perpendicular to the diagonals of the inscribed square.

89. Cut out the circumscribed square and show by folding that it is twice the area of the inscribed square.

90. Construct an equilateral triangle on a horizontal line 1 inch long. On the right side as a base, construct a second equilateral triangle. On the left side of the first triangle, construct a third. Construct three more, completing the hexagon.

91. Can you circumscribe a circle about the above hexagon? What is the radius of the circle?

92. Inscribe a regular hexagon in a circle whose radius is $1\frac{1}{2}$ inch. What is the length of each side of the hexagon?

93. Inscribe in a circle an equilateral triangle. On each of its three sides construct an equilateral triangle.

94. Construct an arc of 60°. Draw two lines meeting at an angle of 60°.

95. Bisect an arc of 60°. Draw two lines meeting at an angle of 30°.

96. Construct an angle of 60° and an angle of 30°. Draw two lines making an angle equal to the sum of the two angles first constructed.

97. Erect a perpendicular at the end of a horizontal line. At the end of a vertical line. At the end of an oblique line.

98. Construct an angle of 45°. An angle of $22\frac{1}{2}$°. An angle of 135°. An angle of 15°. An angle of 75°.

99. Draw a circle, radius 1 inch. Draw a diameter, and produce it an inch beyond the circumference. At the centre of the circle erect a perpendicular to the diameter.

100. An inch from one end of a 3-inch line, erect a perpendicular, using as few and as short lines as possible.

101. Draw a horizontal line. Take a point above the line as a center. Draw an arc that cuts the line in two places.

102. Draw a line. From a point above the line, let fall a perpendicular to the line.

EQUAL TRIANGLES. EQUIVALENT TRIANGLES.

639. NOTE. — The protractor and the triangle may be used in the following exercises.

1. Draw a rectangle, base $2\frac{1}{2}$ inches, altitude 2 inches. Draw a rhomboid, base $2\frac{1}{2}$ inches, altitude 2 inches. Find the area of each.

2. With a base $2\frac{1}{2}$ inches, altitude 2 inches, draw

(*a*) A right-angled triangle.
(*b*) An isosceles triangle.
(*c*) One or more acute-angled scalene triangles.
(*d*) One or more obtuse-angled triangles.

Calculate the area of each.

3. Can you show, by cutting from paper, that a right-angled triangle having its base and perpendicular 4 inches and 3 inches, respectively, has the same surface as an acute-angled triangle whose base and altitude are 4 inches and 3 inches respectively, and an obtuse-angled triangle whose base and altitude are 4 inches and 3 inches, respectively?

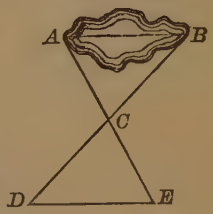
Two triangles that have the same area are called *equivalent* triangles; those having their corresponding sides and angles equal, each to each, are called *equal* triangles.

4. Construct a triangle whose sides measure $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ inches, respectively. Construct another triangle having its sides of the same lengths. Are the angles of the second equal to the angles of the first? Are the triangles equal?

5. Draw two triangles each of which has two sides measuring $1\frac{1}{2}$ and 3 inches, respectively, and the included angle 60 degrees. Is the third side of one triangle equal to the third side of the other? Are the remaining angles of the first triangle equal to the remaining angles of the second?

6. Construct two triangles with equal bases, and angles at the bases respectively equal. Are the triangles equal?

7. A person wishing to ascertain the length, AB, of a pond, places a pole at a convenient point, C, visible from A and B. The distance BC is measured, and a pole is set up, on a line with B and C, at D, the distance CD being made equal to BC. A pole is also placed at E, on a line with A and C, the distance CE being made equal to AC.

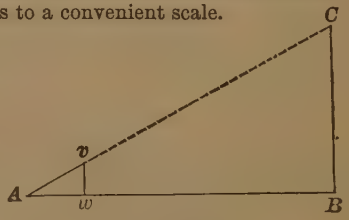


Can you show that the length, AB, of the pond can be ascertained by measuring the distance DE?

CALCULATING HEIGHTS AND DISTANCES.

640. To verify the results obtained by calculation, the pupil should make diagrams, drawing the figures to a convenient scale.

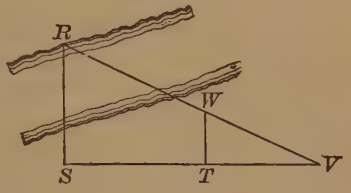
1. If AB in a right-angled triangle measures 120 feet, and a perpendicular, vw, erected 10 feet from A measures 5 feet, calculate the length of BC.



$Aw : AB :: wv : BC$; *i.e.* $10 : 120 :: 5 : BC$.

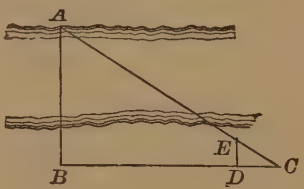
2. A post 6 feet above ground throws a shadow of $7\frac{1}{2}$ feet. How high is a tree whose shadow measures 60 feet?

3. Wishing to ascertain the distance between two houses, R and S, on opposite sides of a stream, I measure a line, SV, at right angles to SR, 200 feet. At T, 90 feet from V, the perpendicular TW measures 60 feet. Required the distance SR.



$$VT : TW :: VS : SR$$

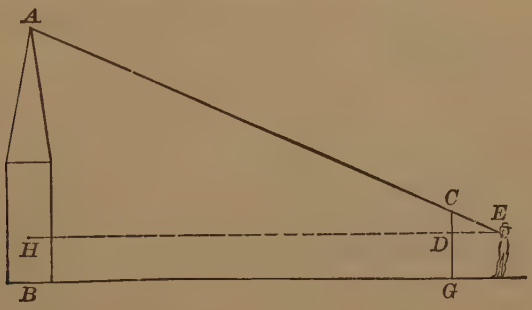
4. Beginning at B, 100 feet from the bank of a river, a line, BC, is measured 1200 feet long. At D, distant from C 50 feet, the perpendicular DE is found to measure 90 feet. What is the distance from B to A, a tree on the opposite bank? How wide is the river?



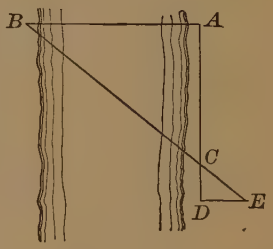
5. A boy, whose eye (E) is 4 feet from the ground, can just see the top (A) of a steeple when he stands back 3 feet from a fence (CG) 6 feet high. The distance from the foot of the fence to the centre of the base of the steeple is 177 feet. Find the height of the steeple, AB.

$$CD = ? \qquad EH = ? \qquad ED : CD :: EH : AH.$$

When AH is found, how may you get AB?

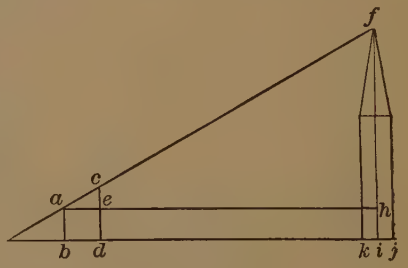


6. Wishing to ascertain the distance AB, I measure a line, AD, at right angles to AB, 12 chains; DE, at right angles to AD, 5 chains; and find that a line sighted from E to B intersects AD at C, distant from D 3.25 chains. What is the distance from A to B?



NOTE. — The triangles DCE and ACB are similar. Why?

7. Wishing to find the height of a tower, fi, I set up a pole, cd, 12 feet long above the ground. Another pole, ab, $4\frac{1}{2}$ feet above ground, is set up at such a distance that the tops of the two poles and of the tower are in a line. The distance between the poles (ae or db) is $10\frac{1}{2}$ feet. The distance from d to the foot of the tower is 195 feet. The width of the tower (kj) is 30 feet.

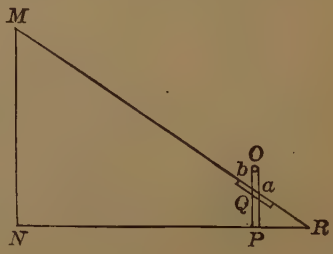


The similar triangles aec and ahf give us the proportion

$$ae : ah :: ec : hf.$$

What is the distance ec? $ah = bi = bd + dk + ki$. $ki = \frac{1}{2}\,kj$. When fh is found, what must be added to get the height of the tower?

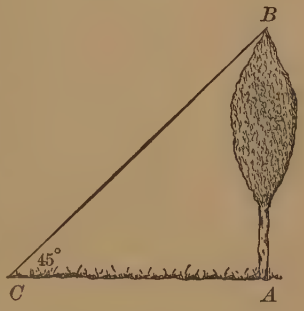
8. To determine the height of a building, MN, a person attached a straight strip of wood, ab, to a post, OP, in such a manner that sighting from a, he could just see M,



the top of the building. He then sighted down from b, and marked on the ground the point R, on a line with ab.

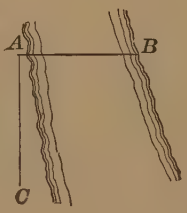
PQ was found by measurement to be 4 feet, RP 6 feet, PN 120 feet. Required, MN.

9. Wood-choppers, desiring to know the height of a tree before cutting it, sometimes make an isosceles right-angled triangle of wood or paper, and "step off" the distance on level ground from the point at which they find they can just see the top of the tree looking along the hypotenuse of the triangle, the base being parallel to the ground.



How high is the tree AB, if AC is 36 paces of 3 feet each, and the angle ACB is 45°?

10. B is a point on the bank of a stream due east of A on the other bank. A boy walks due south of A until he reaches a point at which he finds, from his pocket compass, that he is directly southwest of B. If the distance AC measures 119 yards, how wide is the stream?



TABLES

LINEAR MEASURE

12	inches (in.)	= 1 foot	ft.
3	feet	= 1 yard	yd.
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet	= 1 rod	rd.
40	rods	= 1 furlong	fur.
320	rods	= 1 mile	mi.

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

A *hand*, used in measuring the height of horses, = 4 in. A *knot*, used in measuring distances at sea, = 1.15 mi. A *fathom*, used in measuring the depth of the sea, = 6 ft.

SQUARE MEASURE

144	square inches (sq. in.)	= 1 square foot	sq. ft.
9	square feet	= 1 square yard	sq. yd.
$30\frac{1}{4}$	sq. yd., or $272\frac{1}{4}$ sq. ft.	= 1 square rod	sq. rd.
160	square rods	= 1 acre	A.
640	acres	= 1 square mile	sq. mi.

1 A. = 160 sq. rd. = 4840 sq. yd. = 43,560 sq. ft.

A **Section** of land is a square mile.

Roofing, flooring, and slating are often estimated by the *square*, which contains 100 square feet.

SURVEYORS' MEASURE

In measuring land, surveyors use a chain (ch.) which contains 100 links (l.) and is 4 rods long. Since the chain is 4 rods long, a square chain contains 16 sq. rd., and 10 sq. ch. = 160 sq. rd., or 1 acre.

CUBIC MEASURE

1728	cubic inches (cu. in.)	= 1 cubic foot	cu. ft.
27	cubic feet	= 1 cubic yard	cu. yd.
128	cubic feet	= 1 cord	cd.
16	cubic feet	= 1 cord ft.	cd. ft.
8	cord feet	= 1 cord	cd.

NOTE. — In computing the contents of an enclosing wall, masons and brick-layers regard it as one straight wall whose length is the distance around it on the outside. Corners are thus measured twice.

A *perch* of stone or masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. thick, and 1 ft. high, and contains $24\frac{3}{4}$ cu. ft.

CENTRAL AVENUE PUBLIC SCHOOL NEWARK, N. J.

MEASURES OF CAPACITY

Liquid Measure			Dry Measure		
4 gills	= 1 pint	pt.	2 pints	= 1 quart	qt.
2 pints	= 1 quart	qt.	8 quarts	= 1 peck	pk.
4 quarts	= 1 gallon	gal.	4 pecks	= 1 bushel	bu.

The *standard gallon* contains 231 cubic inches.
The *standard bushel* contains 2150.42 cubic inches.

The capacity of cisterns, reservoirs, etc., is often expressed in barrels (bbl.) of 31½ gallons each, or in hogsheads (hhd.) of 63 gallons each. In commerce, these vary in size.

AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound	lb.
100 pounds	= 1 hundredweight	cwt.
2000 pounds	= 1 ton	T.

One pound Avoirdupois = 7000 grains.

The *long ton* of 2240 pounds is used in the United States Custom Houses and in weighing coal and iron at the mines.

STANDARD WEIGHTS

1 bushel of wheat	= 60 lb.	1 bushel of potatoes	= 60 lb.
1 bushel of corn	= 56 lb.	1 barrel of flour	= 196 lb.
1 bushel of oats	= 32 lb.	1 barrel of pork	= 200 lb.
1 bushel of barley	= 48 lb.	1 keg of nails	= 100 lb.

TROY WEIGHT

24 grains (gr.)	= 1 pennyweight	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

One pound Troy = 5760 grains.

APOTHECARIES' WEIGHT

60 grains (gr.)	= 1 dram	dr., or ℥.
8 drams	= 1 ounce	oz., or ℥.
12 ounces	= 1 pound	lb., or ℔.

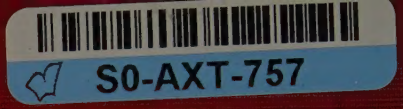
One pound Apothecaries' weight = 5760 grains.

BRITISH OR STERLING MONEY

4 farthings	= 1 penny	*d.*
12 pence	= 1 shilling	*s.*
20 shillings	= 1 pound	£.
5 shillings	= 1 crown.	

The value of £1 is $4.8665 in United States gold coin.

The unit of French money is 1 franc, which is 19.3 cents. The unit of German money is 1 mark, which is 23.85 cents.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